Answer on Question #51420 – Math – Integral Calculus

Question

$\int \frac{x^4}{x^3+x^{-2}}dx = ?$

Solution

$$\int \frac{x^4}{x^3+x^{-2}}dx = \int \frac{x^4}{x^3+\frac{1}{x^2}}dx = \int x^4\frac{x^2}{x^5+1}dx = \int \frac{x^6}{x^5+1}dx = \int \left(x - \frac{x}{x^5+1}\right)dx$$

$$= \frac{x^2}{2} - \int \frac{xdx}{x^5+1} = \frac{x^2}{2} - \int \frac{xdx}{(x+1)(x^4-x^3+x^2-x+1)}$$

$$= \frac{x^2}{2} - \int \frac{xdx}{(x+1)\left(x^2-\frac{\sqrt{5}+1}{2}x+1\right)\left(x^2+\frac{\sqrt{5}-1}{2}x+1\right)} =$$

$$= \frac{x^2}{2} - \int \left(-\frac{1}{5(x+1)} + \frac{x\left(\sqrt{5}+1\right)+\left(1-\sqrt{5}\right)}{5\left(2x^2+\left(\sqrt{5}-1\right)x+2\right)} + \frac{x\left(1-\sqrt{5}\right)+\left(\sqrt{5}+1\right)}{5\left(2x^2-\left(\sqrt{5}+1\right)x+2\right)}\right)dx =$$

$$= \frac{x^2}{2} + \frac{1}{5}\int\frac{dx}{x+1} - \frac{\sqrt{5}+1}{5}\int\frac{xdx}{2x^2+\left(\sqrt{5}-1\right)x+2} - \frac{1-\sqrt{5}}{5}\int\frac{dx}{2x^2-\left(\sqrt{5}+1\right)x+2} - \frac{1-\sqrt{5}}{5}\int\frac{xdx}{2x^2+\left(\sqrt{5}-1\right)x+2} -$$

$$-\frac{\sqrt{5}+1}{5}\int\frac{dx}{2x^2-\left(\sqrt{5}+1\right)x+2} = \frac{x^2}{2} + \frac{1}{5}\ln|x+1| - \frac{\sqrt{5}+1}{20}\ln\left|2x^2+\left(\sqrt{5}-1\right)x+2\right| +$$

$$+\frac{\sqrt{5}-1}{20}\ln\left|-2x^2+\left(\sqrt{5}+1\right)x-2\right| + \frac{2}{\sqrt{5}}\frac{arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}\right)}{\sqrt{10+2\sqrt{5}}} + \frac{2}{\sqrt{5}}\frac{arctan\left(\frac{\sqrt{5}+1-4x}{\sqrt{10-2\sqrt{5}}}\right)}{\sqrt{10-2\sqrt{5}}} + c.$$

Answer: $\frac{x^2}{2} + \frac{1}{5}\ln|x+1| - \frac{\sqrt{5}+1}{20}\ln\left|2x^2+\left(\sqrt{5}-1\right)x+2\right| +$

$+\frac{\sqrt{5}-1}{20}\ln\left|-2x^2+\left(\sqrt{5}+1\right)x-2\right| + \frac{2}{\sqrt{5}}\frac{arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}\right)}{\sqrt{10+2\sqrt{5}}} + \frac{2}{\sqrt{5}}\frac{arctan\left(\frac{\sqrt{5}+1-4x}{\sqrt{10-2\sqrt{5}}}\right)}{\sqrt{10-2\sqrt{5}}} + c.$