Answer on Question #51407 – Math – Calculus

Question

Determine whether each of the following converges or diverges

i)∞

∑ 1/k(k-1)(k-6)

k=10

ii)∞

∑ sin k/ k^2

k=1

iii) ∞

∑ 1/2k^2+k

k=1

iv)∞

∑ 1/(ℓn k)^k

k=2

v) ∞

∑ k^k/ k!

k=1

vi) ∞

∑ (4k-5/2k+1)^k

k=2

Solution

i)

$$\sum_{k=10}^{\infty} \frac{1}{k(k-1)(k-6)}$$

$$\lim_{k\to\infty} \frac{\frac{1}{k(k-1)(k-6)}}{\frac{1}{k^3}} = \lim_{k\to\infty} \frac{k^3}{k(k-1)(k-6)} = 1$$

$\sum_{k=10}^{\infty} \frac{1}{k^3}$ converges as a hyperharmonic series $\sum_{k=1}^{\infty} \frac{1}{k^p}$ with $p = 3 > 1$. So, by the limit comparison test, $\sum_{k=10}^{\infty} \frac{1}{k(k-1)(k-6)}$ converges.

ii)

$$\sum_{k=1}^{\infty} \frac{\sin k}{k^2}$$

$$\left|\frac{\sin k}{k^2}\right| \le \frac{1}{k^2}$$

$\sum_{k=1}^{\infty} \frac{1}{k^2}$ converges as a hyperharmonic series $\sum_{k=1}^{\infty} \frac{1}{k^p}$ with $p = 2 > 1$. So, the series $\sum_{k=1}^{\infty} \left|\frac{\sin k}{k^2}\right|$ converges by the comparison test, hence, $\sum_{k=1}^{\infty} \frac{\sin k}{k^2}$ converges.

iii)

$$\sum_{k=1}^{\infty} \frac{1}{2k^2 + k}$$

$$\lim_{k\to\infty} \frac{\frac{1}{2k^2+k}}{\frac{1}{k^2}} = \lim_{k\to\infty} \frac{k^2}{2k^2+k} = \lim_{k\to\infty} \frac{k}{2k+1} = \frac{1}{2}$$

$\sum_{k=1}^{\infty} \frac{1}{k^2}$ converges as a hyperharmonic series $\sum_{k=1}^{\infty} \frac{1}{k^p}$ with $p = 2 > 1$. So, by the limit comparison test, $\sum_{k=1}^{\infty} \frac{1}{2k^2 + k}$ converges.

iv)

$$\sum_{k=2}^{\infty}\frac{1}{(\ln k)^k}$$

Here we use the root test.

$$\lim_{k\to\infty}\sqrt[k]{\frac{1}{(\ln k)^k}}=\lim_{k\to\infty}\frac{1}{\ln k}=0<1$$

So, $\sum_{k=2}^{\infty}\frac{1}{(\ln k)^k}$ converges.

v)

$$\sum_{k=1}^{\infty}\frac{k^k}{k!}$$

Here we use the root test.

$$\lim_{k\to\infty}\sqrt[k]{\frac{k^k}{k!}}=\lim_{k\to\infty}\frac{k}{\sqrt[k]{k!}}=\frac{\lim\limits_{k\to\infty}k}{\lim\limits_{k\to\infty}\sqrt[k]{k!}}=+\infty>1$$

So, $\sum_{k=1}^{\infty}\frac{k^k}{k!}$ diverges.

vi)

$$\sum_{k=2}^{\infty}\left(\frac{4k-5}{2k+1}\right)^k$$

Let use the root test.

$$\lim_{k\to\infty}\sqrt[k]{\left(\frac{4k-5}{2k+1}\right)^k}=\lim_{k\to\infty}\frac{4k-5}{2k+1}=2>1$$

So, $\sum_{k=2}^{\infty}\left(\frac{4k-5}{2k+1}\right)^k$ diverges.