Answer on Question #51406 – Math – Calculus

1. Calculate:

i) $\lim\limits_{x\to 0}\dfrac{x-tg\,x}{x-\sin x}$; **ii)** $\lim\limits_{x\to 1}\dfrac{3x^2-2x-1}{x^2-x}$; **iii)** $\lim\limits_{x\to 0}\dfrac{e^x-1}{\ln(x+1)}$; **iv)** $\lim\limits_{x\to +0}x^x$.

Solution

i) $$\lim_{x\to 0}\frac{x-tg\,x}{x-\sin x}=\left|\begin{array}{l}\sin x\sim x-\frac{x^3}{3!}\\ tgx\sim x+\frac{x^3}{3}\\ x\to 0\end{array}\right|=\lim_{x\to 0}\frac{x-\left(x+\frac{x^3}{3}\right)}{x-\left(x-\frac{x^3}{6}\right)}=\lim_{x\to 0}\frac{-\frac{x^3}{3}}{\frac{x^3}{6}}=\lim_{x\to 0}(-2)=-2;$$

ii) $$\lim_{x\to 1}\frac{3x^2-2x-1}{x^2-x}=\lim_{x\to 1}\frac{3(x-1)\left(x-\frac{1}{3}\right)}{x(x-1)}=\lim_{x\to 1}\frac{3\left(x-\frac{1}{3}\right)}{x}=\lim_{x\to 1}\frac{3x-1}{x}=\frac{3\cdot 1-1}{1}=2;$$

iii) $\lim\limits_{x\to 0}\dfrac{e^x-1}{\ln(x+1)}=\lim\limits_{x\to 0}\dfrac{e^x-1}{x}\cdot\dfrac{x}{\ln(x+1)}=\lim\limits_{x\to 0}\dfrac{e^x-1}{x}\cdot\lim\limits_{x\to 0}\dfrac{x}{\ln(x+1)}=1\cdot 1=1$ (the last values of limits are well-known).

iv)

$$\lim_{x\to +0}x^x=\lim_{x\to +0}\exp(x\ln x)=\exp\lim_{x\to +0}\frac{\ln x}{1/x}=\left\{\frac{\infty}{\infty}\right\}=/L'hopital\_rule/\overset{L}{=}\exp\lim_{x\to +0}\frac{1/x}{-1/x^2}=\exp\lim_{x\to +0}(-x)=e^{-0}=1$$

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Answer: i) -2; ii) 2; iii) 1; iv) 1.