Answer on Question #51405 – Math – Calculus

Question

Find the Taylor's Polynomial of degree n about x = c and the remainder for the function f given

i) f(x)=√(3-x) and c=1

ii) f(x)=2x^4 + 3x^3 + 4x^2 + 5x + 6, n=3 and c=1

iii) f(x)=cos x and c=0

Solution

A Taylor series is a series expansion of a function about a point. A one-dimensional Taylor series is an expansion of a real function f(x) about a point x=c is given by

$$f(\mathrm{x}) = \mathrm{f(a)} + \mathrm{f}'(\mathrm{a})(\mathrm{x}-\mathrm{a}) + \frac{f''(\mathrm{a})}{2!}(\mathrm{x}-\mathrm{a})^2 + \frac{f'''(\mathrm{a})}{3!}(\mathrm{x}-\mathrm{a})^3 + \cdots + \frac{f^{(n)}(\mathrm{a})}{\mathrm{n}!}(\mathrm{x}-\mathrm{a})^n + \cdots =$$

$$= \sum_{n=0}^{\infty} \frac{f^{(n)}(\mathrm{a})}{\mathrm{n}!}(\mathrm{x}-\mathrm{a})^n$$

i) $f(\mathrm{x}) = \sqrt{3-x} = (3-\mathrm{x})^{1/2} \quad f(1) = \sqrt{2}$

$$f'(\mathrm{x}) = -\frac{1}{2}(3-\mathrm{x})^{-1/2} \quad f'(1) = -\frac{1}{2}(2)^{-1/2}$$

$$f''(\mathrm{x}) = -\frac{1}{4}(3-\mathrm{x})^{-3/2} \quad f''(1) = -\frac{1}{4}(2)^{-3/2}$$

$$f'''(\mathrm{x}) = -\frac{3}{8}(3-\mathrm{x})^{-5/2} \quad f'''(1) = -\frac{3}{8}(2)^{-5/2}$$

$$f^{(4)}(\mathrm{x}) = -\frac{15}{16}(3-\mathrm{x})^{-7/2} \quad f^{(4)}(1) = -\frac{15}{16}(2)^{-7/2}$$

$$\sqrt{3-x} = f(1) + f'(1)(\mathrm{x}-1) + \frac{f''(1)}{2!}(\mathrm{x}-1)^2 + \frac{f'''(1)}{3!}(\mathrm{x}-1)^3 + \cdots =$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^{2n-1}(2\mathrm{n})!}{4^n n!(2\mathrm{n}-1)}(3-\mathrm{c})^{-\frac{2n-1}{2}}(\mathrm{x}-1)^n$$

ii) $f(\mathrm{x}) = 2\mathrm{x}^4 + 3x^3 + 4x^2 + 5x + 6 \quad f(1) = 2+3+4+5+6 = 20$

$$f'(\mathrm{x}) = 8x^3 + 9x^2 + 8x + 5 \quad f'(1) = 8+9+8+5 = 30$$

$$f''(\mathrm{x}) = 24x^2 + 18x + 8 \quad f''(1) = 24+18+8 = 50$$

$$f'''(\mathrm{x}) = 48\mathrm{x} + 18 \quad f'''(1) = 48+18 = 66$$

$$2x^4+3x^3+4x^2+5x+6 = f(1)+f'(1)(x-1)+\frac{f''(1)}{2!}(x-1)^2+\frac{f'''(1)}{3!}(x-1)^3 =$$

$$= 20+30(x-1)+25(x-1)^2+11(x-1)^3$$

iii) $cos(x) = cos(c) - sin(c)\cdot(x-c) - \frac{1}{2}cos(c)\cdot(x-c)^2 + \frac{1}{6}sin(c)\cdot(x-c)^3 + \cdots$

Since $c=0,\ \sin 0 = 0,\ \cos 0 = 1$, so

$$\cos x = 1-0-\frac{1}{2}x^2+0+\frac{1}{24}x^4+\cdots = 1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}+\cdots = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}x^{2n}$$