Answer on Question #51404 – Math – Calculus

Given:

i) $\int_{-\infty}^{+\infty} \frac{2x}{1+x^2} dx$ **ii)** $\int_{1}^{+\infty} \frac{1}{x^3} dx$

Determine the convergence

Solution:

i)

Function $f(x) = \frac{2x}{1+x^2}$, take function $g(x) = \frac{2}{x}$ so that

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{2x \cdot x}{(1+x^2)2} = \lim_{x\to\infty} \frac{x^2}{1+x^2} = 1.$$

$\int_{1}^{+\infty} g(x)dx = \int_{1}^{+\infty} \frac{2}{x} dx = 2\lim_{x\to\infty}(\ln|x| - \ln 1) = +\infty$, $\int_{-\infty}^{1} g(x)dx = \int_{-\infty}^{1} \frac{2}{x} dx = 2\lim_{x\to-\infty}(\ln 1 - \ln|x|) = -\infty$.

Because $\int_{1}^{+\infty} g(x)dx$ and $\int_{-\infty}^{1} g(x)dx$ are divergent and $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 1$, then $\int_{1}^{+\infty} f(x)dx$ and $\int_{-\infty}^{1} f(x)dx$ are divergent, which gives that

$I = \int_{-\infty}^{+\infty} \frac{2x}{1+x^2} dx = \int_{-\infty}^{1} \frac{2x}{1+x^2} dx + \int_{1}^{+\infty} \frac{2x}{1+x^2} dx$ is also divergent.

Answer: divergent.

ii)

$$I = \int_{1}^{+\infty} \frac{1}{x^3} dx = -\frac{1}{2x^2}\Big|_{1}^{+\infty} = 0 + \frac{1}{2} = \frac{1}{2}$$

Answer: convergent, $I = \frac{1}{2}$.