Answer on Question #51401 – Math – Algorithms | Quantitative Methods

The time versus velocity data of a particle is given in the table below. Use Lagrange's interpolation formula to find the distance moved by a particle and its acceleration at the end of 3 seconds.

t :0 ,1 ,2, 5
v: 2, 3 ,12 ,147

Solution

The interpolation polynomial in the Lagrange form is

$$V(t) = \sum_{i=0}^{n} V_i * l_i(t),$$

where $t_0 = 0$, $t_1 = 1$, $t_2 = 2$, $t_3 = 5$, $v_0 = 2$, $v_1 = 3$, $v_2 = 12$, $v_3 = 147$.

Calculate

$$l_0(t) = \prod_{m=1}^{3} \frac{t - t_m}{t_0 - t_m} = \frac{t-1}{0-1} * \frac{t-2}{0-2} * \frac{t-5}{0-5} = -\frac{1}{10}(t^3 - 8t^2 + 17t - 10);$$

$$l_1(t) = \prod_{m=0, m\neq 1}^{3} \frac{t - t_m}{t_1 - t_m} = \frac{t-0}{1-0} * \frac{t-2}{1-2} * \frac{t-5}{1-5} = \frac{1}{4}(t^3 - 7t^2 + 10t);$$

$$l_2(t) = \prod_{m=0, m\neq 2}^{3} \frac{t - t_m}{t_2 - t_m} = \frac{t-0}{2-0} * \frac{t-1}{2-1} * \frac{t-5}{2-5} = -\frac{1}{6}(t^3 - 6t^2 + 5t);$$

$$l_3(t) = \prod_{m=0,}^{2} \frac{t - t_m}{t_3 - t_m} = \frac{t-0}{5-0} * \frac{t-1}{5-1} * \frac{t-2}{5-2} = \frac{1}{60}(t^3 - 3t^2 + 2t);$$

Velocity

$$V(t) = -\frac{1}{10} * 2 * (t^3 - 8t^2 + 17t - 10) + \frac{1}{4} * 3 * (t^3 - 7t^2 + 10t) - \frac{1}{6} * 12 * \\ * (t^3 - 6t^2 + 5t) + \frac{1}{60} * 147 * (t^3 - 3t^2 + 2t) = t^3 + t^2 - t + 2;$$

Acceleration

$$a(t) = \frac{dV}{dt} = 3t^2 + 2t - 1;$$

Acceleration at the end of 3 seconds

$$a(3) = 3 * 9 + 2 * 3 - 1 = 32;$$

The distance moved by a particle

$S(t) = \int_0^t V(u)du;$

The distance moved by a particle at the end of 3 seconds

$$S(3) = \int_0^3 (t^3 + t^2 - t + 2)dt = \left(\frac{t^4}{4} + \frac{t^3}{3} - \frac{t^2}{2} + 2t\right)\Bigg|_0^3 = 30.75;$$

(integration rule for powers and Newton-Leibnitz formula were applied here).

Answer: Distance = 30.75 and acceleration = 32.