

Answer on Question #51400 – Math – Algorithms | Quantitative Methods

The current in an electric circuit is given by $i = t \sin t$ where t is the time in seconds. Using the bisection method, estimate the time required for the current to reach 1 amp (correct up to 2 decimal places)

Solution

We must solve the equation $t \sin t - 1 = 0$ using the bisection method.

The input for the method is a continuous function f , an interval $[a, b]$, and the function values $f(a)$ and $f(b)$. The function values are of opposite sign (there is at least one zero crossing within the interval). Each iteration performs these steps:

1. Calculate c , the midpoint of the interval, $c = 0.5 * (a + b)$.
2. Calculate the function value at the midpoint, $f(c)$.
3. If convergence is satisfactory (that is, $a - c$ is sufficiently small, or $f(c)$ is sufficiently small), return c and stop iterating.
4. Examine the sign of $f(c)$ and replace either $(a, f(a))$ or $(b, f(b))$ with $(c, f(c))$ so that there is a zero crossing within the new interval.

Let's start with values of $a = 0$ and $b = 2$.

$$f(t) = t \sin t - 1$$

$$f(0) = -1$$

$$f(2) = 0.818595$$

So, for example, on the first iteration we get $f(c_1) = -0.158529$ and so must replace the value of the left endpoint of interval a with $c_1 = 1$ hence narrowing the interval.

Iteration process is represented by the following table:

Iteration	a_n	b_n	c_n	$f(c_n)$
1	0	2	1	-0.158529
2	1	2	1.5	0.496242
3	1	1.5	1.25	0.186231
4	1	1.25	1.125	0.015051
5	1	1.125	1.0625	-0.0718266
6	1.0625	1.125	1.09375	-0.0283617
7	1.09375	1.125	1.10938	-0.00664277
8	1.10938	1.125	1.11719	0.00421152
9	1.10938	1.11719	1.11328	-0.00121128
10	1.11328	1.11719	1.11524	0.0014969

We see that root of the equation $t \sin t - 1 = 0$ is 1.11524 with accuracy up to two decimal places.