Answer on Question #51354 - Math - Calculus

A vertical container with base area measuring $a = 14$ cm by $b = 17$ cm is being filled with identical pieces of candy, each with a volume of $V_c = 50.0\text{mm}^3$ and a mass of $m_c = 0.02\text{g}$. Assume that the volume of the empty spaces between the candies is negligible. If the height of the candies in the container increases at the rate of $\frac{dh}{dt} = 0.28\frac{\text{cm}}{\text{s}}$, at what rate does the mass of the candies in the container increase?

Solution

The density of candies is

$$\rho = \frac{m_c}{V_c} = \frac{0.02\text{g}}{50\text{mm}^3} = \frac{2 \cdot 10^{-5}\text{kg}}{5 \cdot 10^{-8}\text{m}^3} = 400\ \frac{\text{kg}}{\text{m}^3}$$

The cross-section of container is

$$S = a \cdot b$$

The volume of candies in the container

$$V = S \cdot h = a \cdot b \cdot h,$$

where h is the height of the candies in the container.

The mass of the candies in the container is

$$m = V \cdot \rho = a \cdot b \cdot h \cdot \rho$$

The mass of candies in the container increase at the rate

$$\frac{dm}{dt} = \frac{d(\rho \cdot a \cdot b \cdot h)}{dt}$$

Since a, b and ρ are constants, we obtain

$$\frac{dm}{dt} = \rho \cdot a \cdot b\frac{dh}{dt} = 400\frac{\text{kg}}{\text{m}^3} \cdot 0.14\text{m} \cdot 0.17\text{m} \cdot 0.0028\frac{\text{m}}{\text{s}} = 26.656\ \frac{\text{g}}{\text{s}}$$

Answer: $\frac{dm}{dt} = \frac{m_c}{V_c} \cdot a \cdot b\frac{dh}{dt} = 26.656\ \frac{\text{g}}{\text{s}}$.