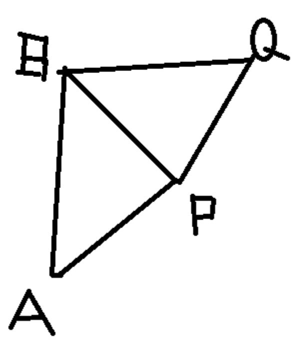
Answer to Question #51111 - Math - Trigonometry

Question

In quadrilateral ABQP, if angle PAB = 60, angle QAB = 45, angle PBA = 30, angle QBA = 75 and AB=20√3, then find the measure of PQ.

Solution



Let's imagine two triangles ABP and ABQ

In triangle ABP we have $\angle APB = 180^{0} - \angle PAB - \angle PBA = 180^{0} - 60^{0} - 30^{0} = 90^{0}$, hence

$AP = AB \cdot \cos 60° = 20\sqrt{3} \cdot \frac{1}{2} = 10\sqrt{3}$; $BP = AB \cdot \sin 60° = 20\sqrt{3} \cdot \frac{\sqrt{3}}{2} = 30.$

In triangle ABQ we have $\angle AQB = 180^{0} - \angle QAB - \angle QBA = 180^{0} - 45^{0} - 75^{0} = 60^{0}$.

Using law of sines, $\frac{AB}{sin\angle AQB} = \frac{BQ}{sin\angle QAB}$, hence

$BQ = \frac{sin\angle QAB}{sin\angle AQB} AB = \frac{sin45°}{sin60°} AB = \frac{1}{\sqrt{2}} \frac{2}{\sqrt{3}} 20\sqrt{3} = 20\sqrt{2}.$

Angle $\angle PBQ = \angle QBA - \angle PBA = 75^{0} - 30^{0} = 45^{0}$.

Using law of cosines,

$PQ^2 = BP^2 + BQ^2 - 2BP \cdot BQ \cdot cos\angle PBQ =$

$= 30^2 + \left(20\sqrt{2}\right)^2 - 2 \cdot 30 \cdot 20\sqrt{2} \cdot cos45^{0} = 500$, hence $PQ = \sqrt{500} \approx 22.4$

Answer: 22.4.