Task 1. *Let R be a field. Prove that the only ideals in R are either $\{0\}$ or R itself.*

Solution. Let $I \subset R$ be an ideal. Suppose $I \neq \{0\}$. Then there is a nonzero $a \in I$. Since $a \neq 0$ and R is a field, $a^{-1} \in R$ is defined. Therefore, $1 = aa^{-1} \in I$, because $a^{-1}I \subset I$ by the definition of an ideal. But then $b = b \cdot 1 \in I$ for all $b \in R$. Thus, $I = R$. □