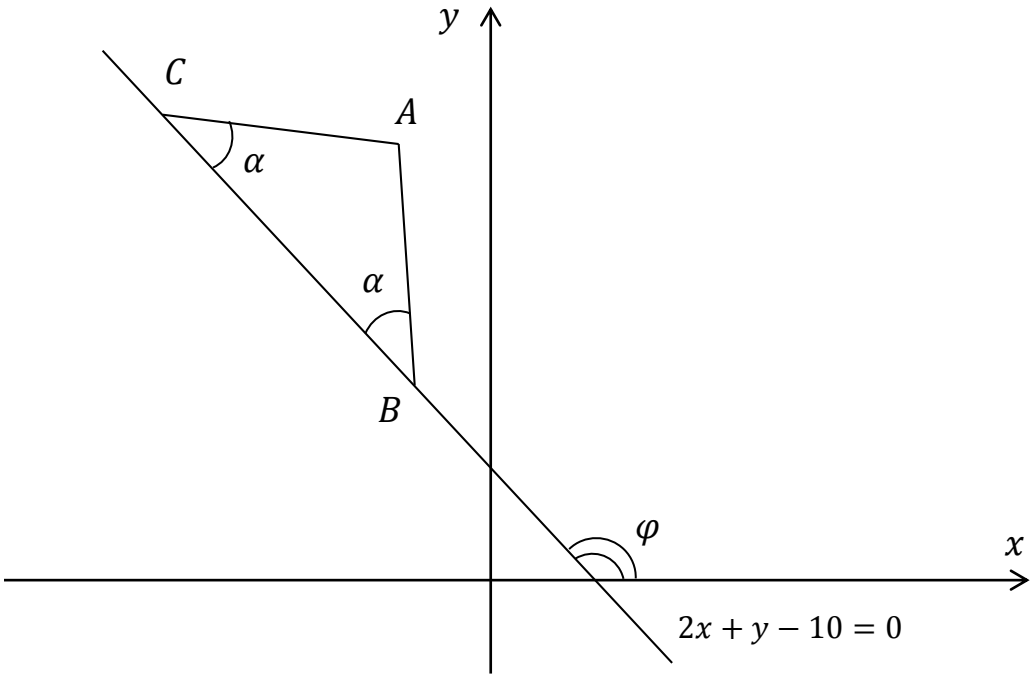
Answer on Question#51031 - Math - Trigonometry

Equation of line BC is $2x + y - 10 = 0$. Point A is such that angle ABC = angle ACB = α = 30°. The sum of the slopes of line AB and line AC is?

Note: Could you please use trigonometry in solving this question?

Solution:



Let's rewrite the equation of line BC in the following way

$$y = 10 - 2x$$

The slope of this line gives us the tangent of the angle which this line creates with x-axis. So

$$\tan\varphi = -2$$

Line AB creates the angle $\varphi - \alpha$ with the x-axis and its slope is

$$\tan(\varphi - \alpha) = \frac{\tan\varphi - \tan\alpha}{1 + \tan\varphi \cdot \tan\alpha} = \frac{-2 - \frac{1}{\sqrt{3}}}{1 - \frac{2}{\sqrt{3}}} = \frac{2\sqrt{3} + 1}{2 - \sqrt{3}}$$

Line AC creates the angle $\varphi + \alpha$ with the x-axis and its slope is

$$\tan(\varphi + \alpha) = \frac{\tan\varphi + \tan\alpha}{1 - \tan\varphi \cdot \tan\alpha} = \frac{-2 + \frac{1}{\sqrt{3}}}{1 + \frac{2}{\sqrt{3}}} = \frac{1 - 2\sqrt{3}}{\sqrt{3} + 2}$$

The sum of slopes of lines AB and AC is

$$\tan(\varphi-\alpha)+\tan(\varphi+\alpha)=\frac{2\sqrt{3}+1}{2-\sqrt{3}}+\frac{1-2\sqrt{3}}{2+\sqrt{3}}=$$
$$=\frac{(2\sqrt{3}+1)(\sqrt{3}+2)+(1-2\sqrt{3})(2-\sqrt{3})}{2^2-\sqrt{3}^2}=16$$

If we consider the case when the point A lies under the line BC the answer will be the same, since the angles which lines AB and AC create with x-axis will be $\varphi+\alpha$ and $\varphi-\alpha$ correspondingly. So the sum of slopes will be the same.

Answer: 16.