

Answer on Question #50973 – Math–Calculus

Question:

Find the area of the bounded plane region R lying between curves $y=x^2 - 2x$ and $y= 4-x^2$.

Solution:

To solve this problem, we need a sketch so that we can determine which function is on top over which intervals. We will begin by determining the points of intersection.

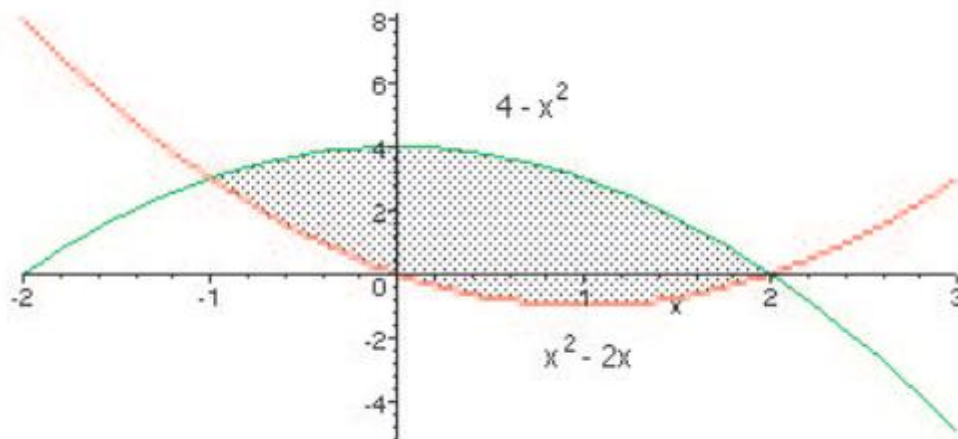
$$x^2 - 2x = 4 - x^2$$

$$2x^2 - 2x - 4 = 0$$

$$2(x^2 - x - 2) = 0$$

$$2(x - 2)(x + 1) = 0$$

So $x = 2$ or $x = -1$. Both functions are quadratic polynomials, so their graphs are parabolas, one opening up and the other down. We should recognize from the sign on x^2 which curve is on top, or we can test a value of x to find out.



Thus, the area of the region can be gotten by applying our integral formula to obtain:

$$\int_{-1}^2 (4 - x^2 - (x^2 - 2x)) dx = \left(4x - \frac{2x^3}{3} + 3x^2 \right) \Big|_{-1}^2 = 8 - \frac{16}{3} + 4 - \left(-4 + \frac{2}{3} + 1 \right) = 9$$