Answer on Question #50896 – Math – Differential Calculus | Equations

Question

A mass m (in kg) acted on by a constant force p Newtons, moves a distance x in t sec. and acquires a velocity $v, \frac{m}{s}$.

Show that:

$$x = \frac{mv^2}{2gp} = \frac{gt^2p}{2m},$$

where g is the acceleration due to gravity.

Solution

Assumptions:

1. At the initial moment of time $(t = 0)$ velocity of the mass is 0.
2. The mass moves only under mentioned above constant force.

Under such assumptions, we can use formulae:

1. $A = Fl$, where F – force, l – distance, A – associated amount of work.
2. $m\ddot{l} = F$, where m – mass, $\ddot{l}$ – acceleration. (Newton's Law)
3. $A = \frac{mv^2}{2}$ (Conservation Law)

Initial conditions:

1. $v(t = 0) = 0$.
2. $l(t = 0) = C$.

Solve differential equation $m\ddot{l} = F$ (second formula):

$\dot{l} = v = \int \frac{F}{m} dt = \frac{F}{m} t + C_1$, because F and m are constants, C_1 is an arbitrary real constant;

$l = \int v dt = \int \left(\frac{F}{m} t + C_1\right) dt = \frac{F}{m}\frac{t^2}{2} + C_1 t + C_2$, where C_1 and C_2 are arbitrary real constants.

Use initial conditions:

$$v(0) = 0, \text{hence } C_1 = 0;$$

$$l(0) = C, \text{hence } C_2 = C.$$

Thus, $l = \frac{F}{m}\frac{t^2}{2} + C$.

Recall that $x = l(t) - l(0)$:

$$x = \frac{F}{m}\frac{t^2}{2} + C - C = \frac{F}{m}\frac{t^2}{2}$$

Equate expressions for A in the first and the third formula:

$$Fl = \frac{mv^2}{2},$$

which yields

$$l = \frac{mv^2}{2F},$$

where l is such that $l(t) - l(0) = x$.

Put values of the quantities into obtained formulae:

$$x = \frac{pt^2}{2m}$$

$$x = \frac{mv^2}{2p}$$