Answer on Question# #50656 – Mathematics – Trigonometry

Question:

Cos^-1 (-x) = ? Please explain the answer.

Solution:

Let us write some definitions.

A function f is said to be an *even* function if for any number x, $f(-x) = f(x)$.

A function f is said to be an *odd* function if for any number x, $f(-x) = -f(x)$.

A function $cos^{-1}(x)$ (or arccos(x), it is usually called arccosine function) is the inverse cosine function, defined to be the inverse of the restricted cosine function $cos(x)$ at interval $0 \leq x \leq \pi$.

Arccosine is neither even nor odd function:

$$arcco\,s(-x) \neq \pm arcco\,s(x).$$

Let us show it:

$$arcco\,s(-x) = arcco\,s(-\cos(\arccos(x))) = arcco\,s(\cos(\pi - \arccos(x))) = \pi - \arccos(x) \qquad \textbf{(1)}$$

Here we used the following relations:

$$\cos(\pi - x) = -\cos(x),$$

$$\cos(\arccos(x)) = x, \qquad when\ -1 \leq x \leq 1,$$

$$\arccos(\cos(y)) = y, \qquad when\ 0 \leq y \leq \pi.$$

Using notation $cos^{-1}(x)$, the left-hand and right-hand sides of (1) give the following equality:

$$cos^{-1}(-x) = \pi - cos^{-1}(x) \qquad \textbf{(2)}$$

Answer: $cos^{-1}(-x) = \pi - cos^{-1}(x)$.