Answer on Question #50654 – Math - Trigonometry

Problem.

2Sin^(-1) x = Sin^(-1) {2x*√(1-x^2)} it's okay. but i think it's proof is if we assume x= sin z?
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$\sin^{-1}$(2sin z*sqrt(1-sin^2 z))
=$\sin^{-1}$(2sin z cos z)
=$\sin^{-1}$(sin 2z)
=2z
=2 $\sin^{-1}$x
2cos^(-1) x = Sin^(-1) {2x*√(1-x^2)} it's also okay

it's my thinking
if we assume x= cos z

$\sin^{-1}$(2cos z*sqrt(1-cos^2 z))
=$\sin^{-1}$(2cos z sin z)
=$\sin^{-1}$(sin 2z)
=2z
=2 $\cos^{-1}$x

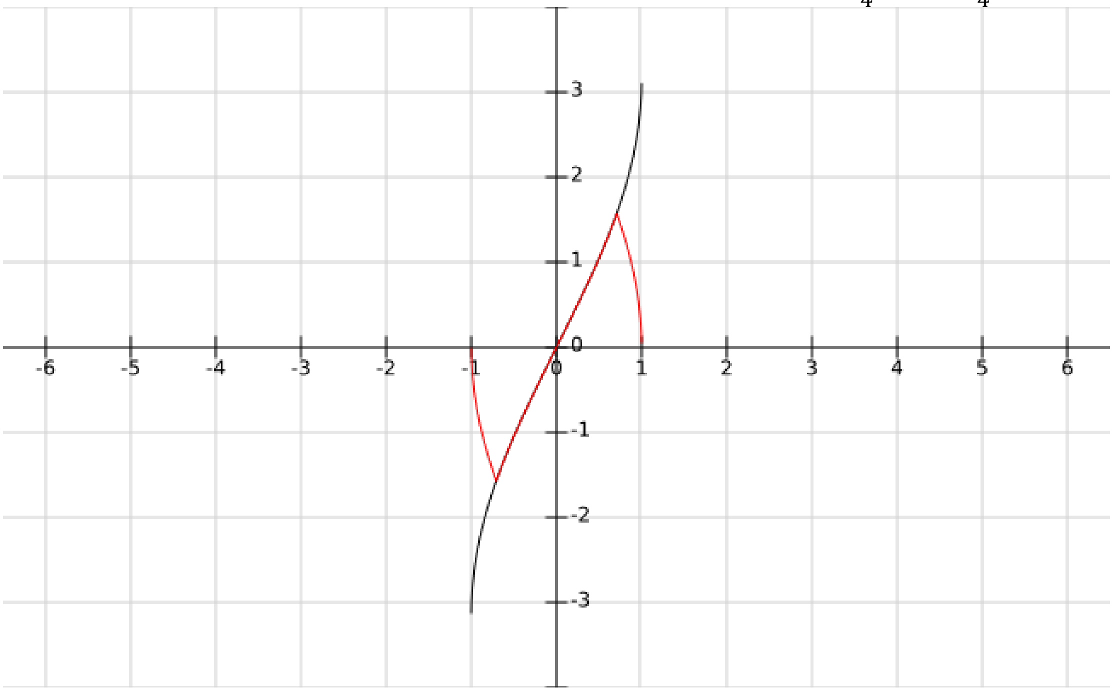
so now my question is are they both correct?
i want to differentiate this Sin^(-1) {2x*√(1-x^2)}
so if we use 2 sin^-1 x , then the answer will be 2/√(1-x^2)

and if we use 2 cos^-1 x , then the answer will be -2/√(1-x^2). there are two different answer after differentiate. please let me know.which one is correct or both correct??

Solution:

In your proof there is a mistake $\sin^{-1}(\sin 2z) = 2z$ only for $-\frac{\pi}{2} \le 2z \le \frac{\pi}{2}$.

Also the formula $2\sin^{-1} x = \sin^{-1} x\sqrt{1-x^2}$ is correct only for $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$.



$y(x) = 2\sin^{-1} x$ black curve, $y(x) = \sin^{-1} x\sqrt{1-x^2}$ red curve.

Also function $\sin^{-1}(2\sin z\sqrt{1-\sin^2 z})$ and $\sin^{-1}(2\cos z\sqrt{1-\cos^2 z})$ are different function of argument x, as in the first case $x = \sin z$ and in the second case $x = \cos z$, so they could have different derivatives and both is true.