Answer on Question #50495 – Math – Trigonometry

If cos(a)= -12/13 and cot(b)=24/7, $90^\circ < a < 180^\circ$ and $180^\circ < b < 270^\circ$ then the quadrant in which $a+b$ lies?

Solution

Since $\cot(b) = \frac{\cos(b)}{\sin(b)}$, then $1+\cot^2(b) = \frac{1}{\sin^2(b)} = 1+\frac{24^2}{7^2} = \frac{625}{49}$, so $\sin^2(b) = \frac{49}{625}$. Due to $180^\circ < b < 270^\circ$ (so, $\sin(b) < 0$ and $\cos(b) < 0$), we obtain $\sin(b) = -\frac{7}{25}$ and $\cos(b) = -\sqrt{1-\sin^2(b)} = -\sqrt{1-\frac{49}{625}} = -\sqrt{\frac{576}{625}} = -\frac{24}{25}$.

Due to $90^\circ < a < 180^\circ$ (so, $\sin(a) > 0$) we obtain $\sin(a) = \sqrt{1-\cos^2(a)} = \sqrt{1-\frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$.

Since $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b) = \frac{5}{13}\cdot\frac{-24}{25} + \frac{-12}{13}\cdot\frac{-7}{25} = \frac{-120+84}{25\cdot 13} = \frac{-36}{25\cdot 13} < 0$ and $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b) = \frac{-12}{13}\cdot\frac{-24}{25} - \frac{5}{13}\cdot\frac{-7}{25} = \frac{288+35}{25\cdot 13} = \frac{323}{25\cdot 13} > 0$ then we obtain that $270^\circ < a+b < 360^\circ$ because there exists only one quadrant where $x = a+b, \cos x > 0$ and $\sin x < 0$ simultaneously.

Answer: $270^\circ < a+b < 360^\circ$, the third quadrant.