Answer on question#50406

Use Mathematical Induction to prove the following:

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n + 1)(2n + 1)}{6}$$

Proof:

For $n = 1$, the statement reduces to $1^2 = \frac{1\cdot 2\cdot 3}{6}$ and is obviously true. Assuming the statement is true for $n = k$:

$$1^2 + 2^2 + 3^2 + \cdots + k^2 = \frac{k(k + 1)(2k + 1)}{6}, \qquad (1)$$

we will prove that the statement must be true for $n = k + 1$:

$$1^2 + 2^2 + 3^2 + \cdots + (k + 1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}. \qquad (2)$$

The left-hand side of (2) can be written as

$1^2 + 2^2 + 3^2 + \cdots + k^2 + (k + 1)^2$.

In view of (1), this simplifies to:

$$1^2 + 2^2 + 3^2 + \cdots + k^2 + (k + 1)^2 = \frac{(k + 1)(k + 2)(2k + 3)}{6} + (k + 1)^2$$
$$= \frac{k(k + 1)(2k + 1) + 6(k + 1)^2}{6}$$
$$= \frac{(k + 1)[k(2k + 1) + 6(k + 1)]}{6} = \frac{(k + 1)(2k2 + 7k + 6)}{6}$$
$$= \frac{(k + 1)(k + 2)(2k + 3)}{6}.$$

Thus the left-hand side of (2) is equal to the right-hand side of (2). This proves the inductive step. Therefore, by the principle of mathematical induction, the given statement is true for every positive integer n.