Answer on Question #50336 – Math – Algorithms | Quantitative Methods

a) Use Newton's method to approximate the root of the equation

xsinx =1 in [0,pi/2].

$$x \sin x = 1, \quad x \in \left[0, \frac{\pi}{2}\right].$$

Taking x=1 as the initial guess calculate x1,x2,x3 (Work to at least 6 decimal places).

b) Show that the equation

$$x^3 - 3x^2 + 2x = 1$$

has no root in the interval [-1,1]

Solution:

a)

According to Newton`s method, we have

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \qquad n = 0,1,2 \ldots$$

where

$$f(x) = x\sin(x) - 1.$$

First of all, we have to find $f'(x)$. So

$$f'(x) = (x\sin(x) - 1)' = \sin(x) + x\cos(x).$$

<u>1 step.</u>

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{1\cdot\sin(1) - 1}{\sin(1) + 1\cdot\cos(1)} \approx 1.1147286724, \quad f'(x_0) \neq 0$$

<u>2 step.</u>

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.1147286724 - \frac{1.1147286724\cdot\sin(1.1147286724) - 1}{\sin(1.1147286724) + 1.1147286724\cdot\cos(1.1147286724)} \approx$$

$$\approx 1.1141571268$$

<u>3 step.</u>

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.1141571268 - \frac{1.1141571268\cdot\sin(1.1141571268) - 1}{\sin(1.1141571268) + 1.1141571268\cdot\cos(1.1141571268)} \approx$$

$$\approx 1.1141571409$$

b)

Denote

$$f(x) = x^3 - 3x^2 + 2x - 1.$$

This function is continuous on $[-1; 1]$.

Any function $f(x)$ that is continuous at each point of a segment $[a; b]$ attains its largest and its smallest values, M and m, on that segment.

Note that the given function is differentiable on $[-1; 1]$.

Let $f(x)$ be continuous on the segment $[a; b]$ and differentiable at all points of this segment (except, possibly, finitely many points in general case). Then the largest and the smallest values of $f(x)$ on $[a; b]$ belong to the set consisting of $f(a), f(b)$, and the values $f(x_i)$, where

$x_i \in (a,\ b)$ are the points at which $f^{'}(x)$ is either equal to zero or does not exist (is infinite).

A function $f(x)$ that is continuous on a segment $[a; b]$ takes any value $c \in [m; M]$ on that segment, where m and M are, respectively, its smallest and its largest values on $[a; b]$.

Thus we have

$$f(-1) = (-1)^3 - 3 \cdot (-1)^2 + 2 \cdot (-1) - 1 = -1 - 3 - 2 - 1 = -7 < 0;$$

$$f(1) = 1^3 - 3 \cdot 1^2 + 2 \cdot 1 - 1 = 1 - 3 + 2 - 1 = -1 < 0.$$

Further more

$$f'(x) = 3x^2 - 6x + 2.$$

We have to solve next equation

$$f'(x) = 0,$$

$$3x^2 - 6x + 2 = 0,$$

$$D = 36 - 4 \cdot 3 \cdot 2 = 36 - 24 = 12 > 0,$$

$$x_{1,2} = \frac{6 \pm \sqrt{12}}{6} = 1 \pm \frac{\sqrt{3}}{2}.$$

Point $x_1 = \left(1 + \frac{\sqrt{3}}{2}\right)$ does not belong to $[-1; 1]$.

So

$$x_2 = \left(1 - \frac{\sqrt{3}}{2}\right) \approx 0.1339745962 \in [-1; 1]$$

and

$$f(x_2) = (0.1339745962)^3 - 3 \cdot (0.1339745962)^2 + 2 \cdot 0.1339745962 - 1 \approx$$

$$\approx -0.7834936491 < 0.$$

Because

$$f(-1) < 0,$$

$$f\left(1-\frac{\sqrt{3}}{2}\right)<0,$$

$$f(1)<0$$

then graph of the function does not cross the OX axis when $x \in [-1,1]$. Thus, the equation

$$x^3-3x^2+2x=1$$

has no roots in the interval [-1,1].

Answer:

a)

$$x_1 = 1.1147286724$$

$$x_2 = 1.1141571268$$

$$x_3 = 1.1141571409$$