Answer on Question #50268 – Math – Complex Analysis

Sequence Zn = [e ^{ i.(n ^2)- (n^2)}] ^ (1/n)

A)) Determine whether Zn is convergent or divergent

Note : please by find the Limit Zn :as n approach to infinity: (if value : the sequence will convergent ,but if Does not exist or infinity the sequence is divergent

B)) is ∑ Zn ^ n Convergent ? Why

Solution

A. $Z_n = \left(\exp\left[in^2 - n^2\right]\right)^{1/n}$

$\lim\limits_{n\to\infty}|Z_n| = \lim\limits_{n\to\infty}\left|\left(\exp\left[in^2 - n^2\right]\right)^{1/n}\right| = \lim\limits_{n\to\infty}\left|\left(\exp\left[-n^2\right]\right)^{1/n}\right| = \lim\limits_{n\to\infty}\left|e^{-n}\right| = 0 < 1$, hence the series is convergent.

B. $Z_n' = (Z_n)^n = \left(\exp\left[in^2 - n^2\right]\right)$

$\lim\limits_{n\to\infty}\left|Z_n{}'\right| = \lim\limits_{n\to\infty}\left|\left(\exp\left[in^2 - n^2\right]\right)\right| = \lim\limits_{n\to\infty}\left|\left(\exp\left[-n^2\right]\right)\right| = \lim\limits_{n\to\infty}\left|e^{-n^2}\right| = 0 < 1$, hence the series is convergent.