A. We have

$$\sum_{n=0}^{\infty} \frac{(-1)^n \left(z-\frac{\pi}{2}\right)^{2n}}{(2n)!}$$

Using the formula for the cosine expansion in a power series $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ is easy to see that

If $z \to \left[z-\frac{\pi}{2}\right]$, than

$$\sum_{n=0}^{\infty} \frac{(-1)^n \left(z-\frac{\pi}{2}\right)^{2n}}{(2n)!} = \cos\left(z-\frac{\pi}{2}\right) = \frac{e^{i\left(z-\frac{\pi}{2}\right)} + e^{-i\left(z-\frac{\pi}{2}\right)}}{2} = \frac{e^{iz}e^{\frac{i\pi}{2}} + e^{-iz}e^{-\frac{i\pi}{2}}}{2}$$

$$= \left\{e^{\frac{i\pi}{2}} = i;\ e^{-\frac{i\pi}{2}} = -i\right\} = \frac{ie^{iz} - ie^{-iz}}{2} = \frac{i}{2}\left(e^{iz} - e^{-iz}\right) = \frac{i}{2}\sum_{n=0}^{\infty} \frac{(1-(-1)^n)i^n}{n!} z^n$$

$$= \frac{i}{2}\sum_{n=0}^{\infty} \frac{2i^{2n+1}}{(2n+1)!} z^{2n+1} = \frac{i}{2}\sum_{n=0}^{\infty} \frac{2i(-1)^n}{(2n+1)!} z^{2n+1} = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{(2n+1)!} z^{2n+1} = \sin z$$

B. We have

$$\sum_{n=0}^{\infty} \frac{1}{(2n)!}$$

Using the formula for the hyperbolical cosine expansion in a power series

$$\sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)!} = \operatorname{csch} z$$

Let z=1, than

$$\sum_{n=0}^{\infty} \frac{1^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{1}{(2n)!} = \operatorname{csch} 1$$

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