Answer on Question #50261 – Math – Complex Analysis

Decide whether the series is convergent or divergent

A. $\sum \frac{8^{n+i2^{-n}}}{9^n}$

B. $\sum \overline{\left(\frac{n+in+n^i}{i^n}\right)}$

Solution

A) $\sum \frac{8^{n+i2^{-n}}}{9^n} = \left(\frac{8}{9}\right)^n 8^{i2^{-n}} \sum \frac{8^{n+i2^{-n}}}{9^n} = \sum \left(\frac{8}{9}\right)^n 8^{i/2^n}.$

By Cauchy criterion

$$q = \lim_{n\to\infty} \sqrt[n]{\left|\left(\frac{8}{9}\right)^n 8^{i/2^n}\right|} = \frac{8}{9} < 1,$$ hence the series is convergent.

B) $\sum \overline{\left(\frac{n+in+n^i}{i^n}\right)} = \sum \overline{\left(\frac{n+in+e^{i\ln n}}{i^n}\right)}$. By Cauchy criterion

$$q = \lim_{n\to\infty} \sqrt[n]{\left|\frac{\overline{n+\imath n+e^{\imath lnn}}}{\imath^n}\right|} = \lim_{n\to\infty} \left|n+in+e^{ilnn}\right|$$

$$= \lim_{n\to\infty} \sqrt{(n+\cos(lnn))^2 + (n+\sin(lnn))^2} = +\infty, \text{hence}$$

the series is divergent.