Answer on Question #50250– Math - Complex Analysis

let ∑ Zn be A convergent series such that 0< Arg (Zn) < (Pi /2)

Show that

∑ (Re (Zn). Im (Zn)) is also Convergent

Solution:

$\sum_{k=1}^{\infty} z_k = X + i \cdot Y$ is the convergent series, where $z_n = x_n + iy_n$. $|X + i \cdot Y| = \sqrt{X^2 + Y^2} < \infty$

If $0 < \arg(Z_n) < \pi / 2$ then $x_n, y_n > 0$.

$$\sum_{k=1}^{\infty} z_k = \sum_{k=1}^{\infty} (x_k + iy_k) = \sum_{k=1}^{\infty} x_k + i \sum_{k=1}^{\infty} y_k = X + i \cdot Y$$

$$\sum_{k=1}^{\infty} \mathrm{Re}[z_k] = \sum_{k=1}^{\infty} x_k = X < \sqrt{X^2 + Y^2} < \infty, \sum_{k=1}^{\infty} \mathrm{Im}[z_k] = \sum_{k=1}^{\infty} y_k = Y < \sqrt{X^2 + Y^2} < \infty$$

$$\left|\sum x_k y_n\right| \le \sum |x_k y_n| \le \sum_{k=1}^{\infty} |x_k| \cdot \sum_{n=1}^{\infty} |y_n|$$

As $x_n, y_n > 0 \Rightarrow \sum |x_k y_n| = \sum x_k y_n$, $\sum_{k=1}^{\infty} |x_k| = \sum_{k=1}^{\infty} x_k = X$, $\sum_{n=1}^{\infty} |y_n| = \sum_{n=1}^{\infty} y_n = Y$, then

$$\sum x_k y_n < X \cdot Y < X^2 + Y^2 < \infty$$

Answer: $\sum \mathrm{Re}[z_k] \mathrm{Im}[z_n]$ is also convergent.