Answer on Question #50191 – Math – Discrete Mathematics

without using truth table prove that
¬(p↔q) and ¬p↔q are logically equivalent.

Solution

Suppose that ¬(p↔q) and ¬p↔q are not logically equivalent.

It means that there exists such p,q that $\overline{p \leftrightarrow q} \neq \bar{p} \leftrightarrow q$.

Consider two cases:

1) $\overline{p \leftrightarrow q} = 0 \Rightarrow p \leftrightarrow q = 1 \Rightarrow p = q \Rightarrow \bar{p} \neq q \Rightarrow \bar{p} \leftrightarrow q = 0$. We have obtained that $\overline{p \leftrightarrow q} = \bar{p} \leftrightarrow q$, because 0=0. It contradicts the assumption $\overline{p \leftrightarrow q} \neq \bar{p} \leftrightarrow q$.
2) $\overline{p \leftrightarrow q} = 1 \Rightarrow p \leftrightarrow q = 0 \Rightarrow p \neq q \Rightarrow \bar{p} = q \Rightarrow \bar{p} \leftrightarrow q = 1$. We have obtained that $\overline{p \leftrightarrow q} = \bar{p} \leftrightarrow q$, because 1=1. It contradicts the assumption $\overline{p \leftrightarrow q} \neq \bar{p} \leftrightarrow q$.

Both cases yield a contradiction. Thus, assumption $\overline{p \leftrightarrow q} \neq \bar{p} \leftrightarrow q$ was wrong, hence ¬(p↔q) and ¬p↔q are logically equivalent.