Answer on Question#50056 - <Math> - <Complex Analysis>

Find Sum of power series, then answer questions

1) $\sum_{n=2}^{\infty}\frac{3^{n+1}z^{n+2}}{n!}$ indicate the convergence neighborhood

Solution. Use the ratio test :

$\lim_{n\to\infty}\frac{\frac{3^{n+2}|z|^{n+3}}{(n+1)!}}{\frac{3^{n+1}|z|^{n+2}}{n!}}=\lim_{n\to\infty}\frac{3|z|}{n+1}=3|z|\lim_{n\to\infty}\frac{1}{n+1}=0<1$, since this inequality holds for any z, then the series $\sum_{n=2}^{\infty}\frac{3^{n+1}z^{n+2}}{n!}$ convergent for any $z\in C$.

Let's find Sum of it:

$$\sum_{n=2}^{\infty}\frac{3^{n+1}z^{n+2}}{n!}=3z^2\sum_{n=2}^{\infty}\frac{3^n z^n}{n!}=3z^2\left(\sum_{n=2}^{\infty}\frac{(3z)^n}{n!}+\frac{(3z)^1}{1!}+\frac{(3z)^0}{0!}-3z-1\right)=3z^2\left(\sum_{n=0}^{\infty}\frac{(3z)^n}{n!}-3z-1\right)=$$
$$=3z^2(e^{3z}-3z-1)$$

Answer: the series convergent for any $z\in C$ and $\sum_{n=2}^{\infty}\frac{3^{n+1}z^{n+2}}{n!}=3z^2(e^{3z}-3z-1)$

2) Is series $\sum_{n=2}^{\infty}\frac{3^{\frac{3n+4}{2}}}{n!}$ convergent, if yes compute its sum

Solution. Use the ratio test

$\lim_{n\to\infty}\frac{\frac{3^{\frac{3n+7}{2}}}{(n+1)!}}{\frac{3^{\frac{3n+4}{2}}}{n!}}=\lim_{n\to\infty}\frac{3^{\frac{3}{2}}}{n+1}=3^{\frac{3}{2}}\lim_{n\to\infty}\frac{1}{n+1}=0<1$, since this inequality holds for any z, then the series $\sum_{n=2}^{\infty}\frac{3^{\frac{3n+4}{2}}}{n!}$ convergent for any $z\in C$.

Let's find Sum of it:

$$\sum_{n=2}^{\infty}\frac{3^{\frac{3n+4}{2}}}{n!}=3^2\sum_{n=2}^{\infty}\frac{\left(3^{\frac{3}{2}}\right)^n}{n!}=3^2\left(\sum_{n=2}^{\infty}\frac{\left(3^{\frac{3}{2}}\right)^n}{n!}+\frac{3^{\frac{3}{2}}}{1!}+1-3^{\frac{3}{2}}-1\right)=3^2\left(\sum_{n=0}^{\infty}\frac{\left(3^{\frac{3}{2}}\right)^n}{n!}-3^{\frac{3}{2}}-1\right)=3^2\left(e^{\sqrt{27}}-\sqrt{27}-1\right)$$

Answer: the series convergent for any $z\in C$ and $\sum_{n=2}^{\infty}\frac{3^{\frac{3n+4}{2}}}{n!}=3^2\left(e^{\sqrt{27}}-\sqrt{27}-1\right)$