50054

Given:

$$\frac{(n+1)^{\sqrt{n+1}}}{n^{\sqrt{n}}}$$

Solution:

1) $$\lim_{n\to\infty}\frac{(n+1)^{\sqrt{n+1}}}{n^{\sqrt{n}}}=\lim_{n\to\infty}\frac{n^{\sqrt{n+1}}+\ldots}{n^{\sqrt{n}}}=\lim_{n\to\infty}n^{\sqrt{n+1}-\sqrt{n}}=\lim_{n\to\infty}n^{\frac{(\sqrt{n+1}-\sqrt{n})(\sqrt{n+1}+\sqrt{n})}{\sqrt{n+1}+\sqrt{n}}}=$$

$$=\lim_{n\to\infty}n^{\frac{n+1-n}{\sqrt{n+1}+\sqrt{n}}}=\lim_{n\to\infty}n^{\frac{1}{\sqrt{n+1}+\sqrt{n}}}=1$$

2) $$\lim_{n\to 0}\frac{(n+1)^{\sqrt{n+1}}}{n^{\sqrt{n}}}=\lim_{n\to 0}\frac{1}{\sqrt[n]{n}}=\lim_{n\to 0}\sqrt[n]{n}=1$$

Answer: $\lim\limits_{n\to\infty}\frac{(n+1)^{\sqrt{n+1}}}{n^{\sqrt{n}}}=1$