# 50047

**Given**:

$$f(z)=\frac{\sin(\frac{1}{z-\pi})}{2z}$$

1) Find the singularities of f , and all possible annulus centered at each singularity
2) Find the laurent series of f in each annulus
3) classify each singularity
4) compute the residue of at each singular point

**Solution**:

1) singularities is those points wherever a function is analytical, so we obtain

$z=0$ isolated singularity

$z=\pi$ isolated singularity

$z=\infty$ singularity

2) the Laurent series in each annulus is

$$f(z)=\frac{\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}z^{2n+1}}{2z}=\frac{1}{2}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}z^{2n}$$

3) classifying of each singularity

a) $z=0$

$$\lim_{z\to 0}\frac{\sin(\frac{1}{z-\pi})}{2z}=\infty \qquad \Rightarrow \qquad z=0 \quad \text{simple pole}$$

b) $z=\pi$

$$\lim_{z\to \pi}\frac{\sin(\frac{1}{z-\pi})}{2z}=\frac{1}{2\pi}\sin(\frac{1}{0}) \quad \nexists \qquad \Rightarrow \qquad z=\pi \quad \text{substantially singularity}$$

c) $z=\infty$

$$\lim_{z\to \infty}\frac{\sin(\frac{1}{z-\pi})}{2z}=0 \qquad \Rightarrow \qquad z=\infty \qquad \text{removable singularity}$$

4) the residue at each singular point

a) $z=0$

$$\operatorname*{Res}_{z=0} f(z)=\lim_{z\to 0}(z-0)^1\cdot f(z)=\lim_{z\to 0} z\cdot\frac{\sin(\frac{1}{z-\pi})}{2z}=\frac{1}{2}\sin(-\frac{1}{\pi})$$

b) $z = \pi$

$\operatorname*{Res}_{z=\pi} f(z) = c_{-1} = 0$ because $z = \pi$ substantially singularity

c) $z = \infty$

$$\operatorname*{Res}_{z=\infty} f(z) = -(\operatorname*{Res}_{z=0} f(z) + \operatorname*{Res}_{z=\pi} f(z)) = -\frac{1}{2}\sin(-\frac{1}{\pi})$$