Answer on Question #49963 - Math - Complex Analysis

Use the Maclaurin series of $f(z)=\cos^2 z-\sin^2 z$ to compute integral on Curve for $\int\limits_{|z|=2}\frac{\cos^2 z-\sin^2 z}{z^{53}}dz$

Solution.

Let's consider $f(z)=\cos^2 z-\sin^2 z=\cos 2z$ then the Maclaurin series for it $f(z)=\cos 2z=\sum\limits_{k=0}^{\infty}(-1)^k\frac{(2z)^{2k}}{(2k)!}$.

Thus, $\int\limits_{|z|=2}\frac{\cos^2 z-\sin^2 z}{z^{53}}dz=\int\limits_{|z|=2}\frac{\cos 2z}{z^{53}}dz=2\pi i\quad res\frac{\cos 2z}{z^{53}}=2\pi i(-1)^{26}\frac{2^{52}}{(52)!}=\pi i\frac{2^{53}}{(52)!}$

Answer: $\int\limits_{|z|=2}\frac{\cos^2 z-\sin^2 z}{z^{53}}dz=\pi i\frac{2^{53}}{(52)!}$