49962

1) **Given**:

$$f(z)=\sum_{n=1}^{\infty}\frac{(z+5i)^n}{n^{\sqrt{n}}}$$

Solution:

$$c_n=\frac{1}{n^{\sqrt{n}}}$$

R is a radius of convergence
U_R is the disk of convergence

$$R=\frac{1}{\overline{\lim_{n\to\infty}}\sqrt[n]{\left|\frac{1}{n^{\sqrt{n}}}\right|}}=\frac{1}{\overline{\lim_{n\to\infty}}\frac{1}{n^{\frac{1}{\sqrt{n}}}}}=1$$

then $U_R=\{z:|z+5i|<1\}$

Answer: $R=1$
$U_R=\{z:|z+5i|<1\}$

2) **Given**:

$$f(z)=\sum_{n=1}^{\infty}\frac{z^n}{n(n+i)(n+2)}$$

Solution:

at first we compute $|n+i|=\sqrt{n^2+1}$

$$R=\frac{1}{\overline{\lim_{n\to\infty}}\sqrt[n]{\left|\frac{1}{n(n+i)(n+2)}\right|}}=\frac{1}{\overline{\lim_{n\to\infty}}\frac{1}{\sqrt[n]{n}\sqrt[n]{n^2+1}\sqrt[n]{n+2}}}=1$$

and $U_R=\{z:|z|<1\}$

Answer: $R=1$
$U_R=\{z:|z|<1\}$