Answer on Question #49961 – Math – Complex Analysis

1) Given:

$$\sum_{n=1}^{\infty}(-1)^{n+1}n(z-1)^n$$

Solution:

$c_n = (-1)^{n+1}n$ is the nth term

R is a radius of convergence
U_R is the convergence neighborhood

$$R = \frac{1}{\lim\limits_{n\to\infty}\sqrt[n]{|c_n|}} = \frac{1}{\lim\limits_{n\to\infty}\sqrt[n]{|n|}} = 1$$

then $U_R = \{z : |z-1| < 1\}$

Answer: $R = 1$
$U_R = \{z : |z-1| < 1\}$

2) Given:

$$\sum_{n=1}^{\infty} n(1-4i)^n$$

Solution:

Assume that the initial series is convergent.

The series $\sum_{n=1}^{\infty} n(1-4i)^n = (1-4i)\sum_{n=1}^{\infty} n(1-4i)^{n-1}$ is the derivative of $(1-4i)\sum_{n=1}^{\infty}(1-4i)^n$.

Convergent series are differentiable. The last series is convergent only if $|1-4i| < 1$, which is false, because $|1-4i| = \sqrt{17} > \sqrt{16} = 4$, i.e. $|1-4i| > 1$. We have obtained a contradiction, our assumption was not correct. Thus, the series $\sum_{n=1}^{\infty} n(1-4i)^n$ diverges.

Answer: not convergent