Answer on Question #49960 – Math – Complex Analysis

1) Given:

$$\sum_{n=1}^{\infty} z^{n+2} 3^{n+1}$$

Solution:

$$\sum_{n=1}^{\infty} z^{n+2} 3^{n+1} = \frac{1}{3}\sum_{n=1}^{\infty} z^{n+2} 3^{n+2} = \frac{1}{3}\sum_{n=3}^{\infty} z^{n} 3^{n}$$

$$c_n = 3^n$$

R is a radius of convergence

U_R is the convergence neighborhood

$$R = \frac{1}{\overline{\lim\limits_{n\to\infty}}\sqrt[n]{\left|3^n\right|}} = \frac{1}{\lim\limits_{n\to\infty}\sqrt[n]{3^n}} = \frac{1}{3}$$

Then the convergence neighborhood is $U_R = \left\{ z : |z| < \frac{1}{3} \right\}$

Answer: $R = \frac{1}{3}$

$$U_R = \left\{ z : |z| < \frac{1}{3} \right\}$$

2) Given:

$$\sum_{n=2}^{\infty} \frac{3^{\frac{3n+4}{2}}}{n!}$$

Solution:

$a_n = \dfrac{9 \cdot 3^{\frac{3n}{2}}}{n!}$ $\qquad \dfrac{a_{n+1}}{a_n} = \dfrac{9 \cdot 3^{\frac{3n+3}{2}}}{(n+1)\cdot n!} \cdot \dfrac{n!}{9 \cdot 3^{\frac{3n}{2}}} = \dfrac{\sqrt{27}}{n+1} < 1$ when $n \to \infty$, so the series is convergent.

The sum of series is

$$\sum_{n=2}^{\infty} \frac{3^{\frac{3n+4}{2}}}{n!} = 3^2 \sum_{n=2}^{\infty} \frac{\left(3^{3/2}\right)^n}{n!} = 9\sum_{n=2}^{\infty} \frac{\left(\sqrt{27}\right)^n}{n!} = 9\left(\sum_{n=0}^{\infty} \frac{\left(\sqrt{27}\right)^n}{n!} - 1 - \sqrt{27} \right) = 9\left(e^{\sqrt{27}} - 1 - \sqrt{27}\right).$$

We used Maclaurin series of function e^t .

Answer: $S = 9\left(e^{\sqrt{27}} - 1 - \sqrt{27}\right)$