Answer on Question#49959 - Math -Complex Analysis.

Use Maclauin series of e^z to compute series from 0 to ∞ of $\frac{\cos(\frac{n\varphi}{3})}{n!}$.

Solution. Let φ be real and $z = e^{i\varphi/3}$. Maclaurin for e^z is $e^z = \sum_{k=0}^{\infty} \frac{z^k}{k!}$. Our sum is $\sum_{k=0}^{\infty} \frac{\cos(\frac{n\varphi}{3})}{n!} = \sum_{k=0}^{\infty} Re\frac{z^n}{n!} = Re\sum_{k=0}^{\infty} \frac{z^n}{n!} = Re(e^z) = Re\left(e^{\cos\left(\frac{\varphi}{3}\right)+isin\left(\frac{\varphi}{3}\right)}\right) =$
$Re(e^{\cos\left(\frac{\varphi}{3}\right)}(\cos\left(\sin\left(\frac{\varphi}{3}\right) + isin\left(\frac{\varphi}{3}\right)\right) = e^{\cos\left(\frac{\varphi}{3}\right)} \cos(\sin\left(\frac{\varphi}{3}\right)).$

Answer: $e^{\cos\left(\frac{\varphi}{3}\right)} \cos\left(\sin\left(\frac{\varphi}{3}\right)\right).$