Answer to Question # 49958

Find Taylor series of the function:

1) $f(z)=4z^3+2z^2-z-5$, $z_0=-1$

Solution:

$$f(z)=\sum_{k=0}^{n}\frac{f^{(k)}(z_0)}{k!}(z-z_0)+\overline{\overline{o}}((z-z_0)^n)=\sum_{k=0}^{3}\frac{f^{(k)}(-1)}{k!}(z+1)^k$$

At first we need to compute the derivatives:

$$f^{(0)}(z)=f(z)=4z^3+2z^2-z-5$$
$$f^{(1)}(z)=f'(z)=12z^2+4z-1$$
$$f^{(2)}(z)=f''(z)=24z+4$$
$$f^{(3)}(z)=f'''(z)=24$$

So the solution is:

$$f(z)=\sum_{k=0}^{3}\frac{f^{(k)}(-1)}{k!}(z+1)^k=[4\cdot(-1)^3+2\cdot(-1)^2+1-5]+[12\cdot(-1)^2+4\cdot(-1)-1](z+1)+$$
$$+[24\cdot(-1)+4]\frac{(z+1)^2}{2}+24\cdot\frac{(z+1)^3}{6}=4(z+1)^3-10(z+1)^2+7(z+1)-6$$

Answer: $4(z+1)^3-10(z+1)^2+7(z+1)-6$

2) $f(z)=\sqrt[3]{e^{z-2}}$, $z_0=i$

Solution:

$$f(z)=\{new\_\mathrm{var}\ iable\_t=z-i\}=\sqrt[3]{e^{t+i-2}}=e^{\frac{t+i-2}{3}}=e^{\frac{i-2}{3}}\cdot e^{\frac{t}{3}}=e^{\frac{i-2}{3}}\cdot\sum_{n=0}^{\infty}\frac{(t/3)^n}{n!}=$$
$$=e^{\frac{i-2}{3}}\cdot\sum_{n=0}^{\infty}\frac{(\frac{z-i}{3})^n}{n!}=\frac{e^{\frac{i}{3}}}{\sqrt[3]{e^2}}\cdot\sum_{n=0}^{\infty}\frac{(\frac{z-i}{3})^n}{n!}$$

Answer: $\frac{e^{\frac{i}{3}}}{\sqrt[3]{e^2}}\cdot\sum_{n=0}^{\infty}\frac{(\frac{z-i}{3})^n}{n!}$

3) $f(z) = e^z Cosh(z)$, $z_0 = 0$

Solution:

we know that

$$Cosh(z) = \frac{e^z + e^{-z}}{2}$$

so we obtain:

$$f(z) = e^z \cdot \frac{e^z + e^{-z}}{2} = \frac{e^{2z}}{2} + \frac{1}{2} = \frac{1}{2} + \frac{1}{2}\sum_{n=0}^{\infty}\frac{(2z)^n}{n!}$$

Answer: $\frac{1}{2} + \frac{1}{2}\sum_{n=0}^{\infty}\frac{(2z)^n}{n!}$