Task 1. *Suppose that the limit as n approaches infinity of x_n equals 0. If $\{y_n\}$ is a bounded sequence, prove that the limit as n approaches infinity of $x_n y_n$ equals 0.*

Solution. Let $\varepsilon > 0$ be a positive real number. Find $N \in \mathbb{N}$ such that $|x_n y_n| < \varepsilon$ for all $n > N$.

Indeed, since y_n is bounded, there is $M \geq 0$ such that $|y_n| \leq M$ for all $n \in \mathbb{N}$. Furthermore, $x_n \to 0$, therefore, there is $N \in \mathbb{N}$ such that $|x_n| < \frac{\varepsilon}{M}$ for all $n > N$. Then

$$|x_n y_n| = |x_n| \cdot |y_n| < \frac{\varepsilon}{M} \cdot M = \varepsilon$$

for all $n > N$. Since ε was an arbitrary positive number, the last means that $x_n y_n \to 0$ as $n \to \infty$. □