Answer on Question #49791 – Math –Matrix | Tensor Analysis

$$\begin{matrix} 4x & + & 2y & + & 3z & = & 35 \\ x & + & 3y & + & 2z & = & 45 \\ 2x & + & y & + & 5z & = & 28 \end{matrix}$$

Calculate $x, y,\ z$ by Cramer's Rule.

Solution:

Coefficient matrix

$$\begin{pmatrix} 4 & 2 & 3 \\ 1 & 3 & 2 \\ 2 & 1 & 5 \end{pmatrix}$$

And additional column

$$\begin{pmatrix} 35 \\ 45 \\ 28 \end{pmatrix}$$

We have the left-hand side of the system with the variables (the "coefficient matrix") and the right-hand side with the answer values.

Let D be the determinant of the coefficient matrix of the above system, and let D_x be the determinant formed by replacing the $x-$column values with the answer-column values

$$D = \begin{vmatrix} 4 & 2 & 3 \\ 1 & 3 & 2 \\ 2 & 1 & 5 \end{vmatrix}$$

$$D_x = \begin{vmatrix} 35 & 2 & 3 \\ 45 & 3 & 2 \\ 28 & 1 & 5 \end{vmatrix}$$

Similarly, then D_y and D_z would be as follows:

$$D_y = \begin{vmatrix} 4 & 35 & 3 \\ 1 & 45 & 2 \\ 2 & 28 & 5 \end{vmatrix}$$

$$D_z = \begin{vmatrix} 4 & 2 & 35 \\ 1 & 3 & 45 \\ 2 & 1 & 28 \end{vmatrix}$$

Evaluating each determinant, we get:

$$D = \begin{vmatrix} 4 & 2 & 3 \\ 1 & 3 & 2 \\ 2 & 1 & 5 \end{vmatrix} = expand\ the\ determinant\ by\ minors\ using\ the\ first\ column =$$

$$= 4\begin{vmatrix} 3 & 2 \\ 1 & 5 \end{vmatrix} - \begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix} + 2\begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} =$$

$$= 4 * (3 * 5 - 1 * 2) - (2 * 5 - 1 * 3) + 2(2 * 2 - 3 * 3)$$

$$= 4 * (15 - 2) - (10 - 3) + 2 * (4 - 9) = 52 - 7 - 10 = 35$$

$$D_x = \begin{vmatrix} 35 & 2 & 3 \\ 45 & 3 & 2 \\ 28 & 1 & 5 \end{vmatrix} = expand\ the\ determinant\ by\ minors\ using\ the\ first\ column$$

$$= 35 * (15 - 2) - 45 * (10 - 3) + 28 * (4 - 9) = 455 - 315 - 140 = 0$$

$$D_y = \begin{vmatrix} 4 & 35 & 3 \\ 1 & 45 & 2 \\ 2 & 28 & 5 \end{vmatrix} = expand\ the\ determinant\ by\ minors\ using\ the\ first\ column$$

$$= 4 * (225 - 56) - 1 * (175 - 84) + 2(70 - 135) = 676 - 91 - 130 = 455$$

$$D_z = \begin{vmatrix} 4 & 2 & 35 \\ 1 & 3 & 45 \\ 2 & 1 & 28 \end{vmatrix} = expand\ the\ determinant\ by\ minors\ using\ the\ first\ column$$

$$= 4 * (84 - 45) - 1 * (56 - 35) + 2 * (90 - 105) = 156 - 21 - 30 = 105$$

Cramer's Rule says that $x = \frac{D_x}{D}$, $y = \frac{D_y}{D}$, $z = \frac{D_z}{D}$

That is

$$x = \frac{0}{35} = 0$$

$$y = \frac{455}{35} = 13$$

$$z = \frac{105}{35} = 3$$

Answer: $\{\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}\} = \{\mathbf{0}, \mathbf{13}, \mathbf{3}\}$