Answer on Question #49789 – Math – Matrix | Tensor Analysis

$$\begin{matrix} x & + & y & + & 2z & = 5 \\ 2x & - & 3y & + & 5z & = 5 \\ x & + & 2y & - & z & = 6 \end{matrix}$$

Solve by matrix. And also calculate $x, y,\ z$.

Solution:

The first step is to turn three variable system of equations into a 3x4 Augmented matrix.

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ 2 & -3 & 5 & 5 \\ 1 & 2 & -1 & 6 \end{array}\right)$$

Next we label the rows of the matrix:

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ 2 & -3 & 5 & 5 \\ 1 & 2 & -1 & 6 \end{array}\right)\begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$$

Next we need to change all the entries below the leading coefficient of the first row to zeros.

For the second row, we can achieve this by multiplying by $-\frac{1}{2}$ and then adding the result to row 1. For the third row, we can multiply by -1 and then also add the result to row 1.

$$\begin{matrix} \\ -\frac{1}{2}R_2 \\ -1R_3 \end{matrix}\left(\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ -1 & \frac{3}{2} & -\frac{5}{2} & -\frac{5}{2} \\ -1 & -2 & 1 & -6 \end{array}\right)\begin{matrix} R_1' \\ R_2 \\ R_3 \end{matrix}$$

$$\begin{matrix} \\ -\frac{1}{2}\mathrm{R}_2 + \mathrm{R}_1 \\ -1\mathrm{R}_3 + \mathrm{R}_1 \end{matrix}\left(\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ 0 & \frac{5}{2} & -\frac{1}{2} & \frac{5}{2} \\ 0 & -1 & 3 & -1 \end{array}\right)\begin{matrix} \mathrm{R}_1' \\ \mathrm{R}_2' \\ \mathrm{R}_3' \end{matrix}$$

We need the leading element in the second row to also be one. We obtain this result by multiplying the second row by $\frac{2}{5}$.

$$\begin{matrix} \\ \frac{2}{5}R_2' \\ \\ \end{matrix}\left(\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ 0 & 1 & -\frac{1}{5} & 1 \\ 0 & -1 & 3 & -1 \end{array}\right)\begin{matrix} R_1' \\ R_2'' \\ R_3' \end{matrix}$$

Next we zero out the element in row three beneath the leading coefficient in row two. For achieve these adding row two and three.

$$\begin{matrix} \\ \\ R_2'' + R_3' \end{matrix}\left(\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ 0 & 1 & -\frac{1}{5} & 1 \\ 0 & 0 & \frac{14}{5} & 0 \end{array}\right)\begin{matrix} R_1' \\ R_2'' \\ R_3'' \end{matrix}$$

From the above matrix, we solve for the variables starting with z in the last row

$$\frac{14}{5}z = 0$$

So $z = 0$

Next we solve for y by substituting for z in the equation formed by the second row:

$$y - \frac{1}{5}z = 1$$

$$y = 1 + 0$$

$$y = 1$$

Finally we solve for x by substituting the values of y and z into the equation formed by the first row:

$$x + y + 2z = 5$$

$$x = 5 - 1 - 0$$

$$x = 4$$

Therefore, the solution to the system of equations is $\{x, y, z\} = \{4,1,0\}$

Answer: $\{4,1,0\}$