Answer on Question #49487 - Math - Complex Analysis

1) $\sum_{n=1}^{\infty}\frac{3i+n}{n^3+n+1}$ — test for convergence

Solution. Test it for absolute convergence:

$\sum_{n=1}^{\infty}\left|\frac{3i+n}{n^3+n+1}\right|=\sum_{n=1}^{\infty}\frac{\sqrt{n^2+9}}{n^3+n+1}$, use the comparison test $0\le\frac{\sqrt{n^2+9}}{n^3+n+1}\le\frac{\sqrt{10n^2}}{3n^3}=\frac{\sqrt{10}}{3n^2}$, but the series $\sum_{n=1}^{\infty}\frac{\sqrt{10}}{3n^2}$ converges because the power of n in denominator greater than 1. So, $\sum_{n=1}^{\infty}\frac{3i+n}{n^3+n+1}$ is absolutely convergent.

Answer: the series is absolutely convergent.

2) $\sum_{n=1}^{\infty}\frac{n+i}{4^n}$ — test for convergence

Solution. Test it for absolute convergence:

$\sum_{n=1}^{\infty}\left|\frac{n+i}{4^n}\right|=\sum_{n=1}^{\infty}\frac{\sqrt{n^2+1}}{4^n}$, use the ratio test $\lim_{n\to\infty}\frac{\frac{\sqrt{(n+1)^2+1}}{4^{n+1}}}{\frac{\sqrt{n^2+1}}{4^n}}=\lim_{n\to\infty}\frac{\sqrt{(n+1)^2+1}}{4\sqrt{n^2+1}}=\lim_{n\to\infty}\frac{n}{n}\frac{\sqrt{(1+\frac{1}{n})^2+\frac{1}{n^2}}}{4\sqrt{1+\frac{1}{n^2}}}=\frac{1}{4}<1$, that is why the series $\sum_{n=1}^{\infty}\frac{\sqrt{n^2+1}}{4^n}$ converges. So, $\sum_{n=1}^{\infty}\frac{n+i}{4^n}$ is absolutely convergent.

Answer: the series is absolutely convergent.

3) $\sum_{n=1}^{\infty}\left(\frac{1}{n^i}\right)^2$ — test for convergence

Solution. Let us check the vanishing condition:

$\lim_{n\to\infty}\left|\left(\frac{1}{n^i}\right)^2\right|=\lim_{n\to\infty}\left|\left(\frac{1}{e^{iLn(n)}}\right)^2\right|=\lim_{n\to\infty}\left|e^{-i2Ln(n)}\right|=\lim_{n\to\infty}\left|e^{-i2(\ln n+i2\pi k)}\right|=\lim_{n\to\infty}\left|e^{4\pi k}e^{-i2\ln n}\right|=e^{4\pi k}\ne 0$, where k is a integer constant. Vanishing condition is the necessary condition for summability. So, the series $\sum_{n=1}^{\infty}\left(\frac{1}{n^i}\right)^2$ diverges.

Answer: the series diverges.

4) $\sum_{n=1}^{\infty}e^{i\cosh n}$ — test for convergence

Solution. First of all, lets consider $e^{i\cosh n}$, n is a integer number, then $\cosh n=\frac{e^n+e^{-n}}{2}$ is real number, this mean that $\left|e^{i\cosh n}\right|=1$

Let us check the vanishing condition:

$\lim_{n\to\infty}\left|e^{i\cosh n}\right| = 1 \neq 0$. Vanishing condition is the necessary condition for summability. So, the series $\sum_{n=1}^{\infty} e^{i\cosh n}$ diverges.

Answer: the series diverges.