Answer on Question #49485 – Math – Complex Analysis

Test the series for convergence : Details

1) [i^n] / [2^(n+2)]

2) [n! ^ 2] / [e^n]

3) [1] / [{square root of (i+n) }^n]

4) conjugate [(1 / (n^i)]

Solution

1) $\left|\frac{i^n}{2^{n+2}}\right| = \frac{1}{2^{n+2}} = \frac{1}{4}\left(\frac{1}{2}\right)^n$ is a geometric sequence with common ratio $q = \frac{1}{2} < 1$, so the series is convergent.

2) $\frac{c_{n+1}}{c_n} = \frac{\left((n+1)!\right)^2}{e^{n+1}} : \frac{(n!)^2}{e^n} = \frac{(n+1)^2}{e} > 1$ for $n \geq 1$, $\frac{c_{n+1}}{c_n} = \frac{(n+1)^2}{e} \to \infty$ as $n \to \infty$. By d`Alembert`s ratio test, the series $\sum_{n=1}^{\infty} \frac{(n!)^2}{e^n}$ diverges.

3) $\left|\frac{1}{\left(\sqrt{i+n}\right)^n}\right| = \frac{1}{|i+n|^{\frac{n}{2}}} < \frac{1}{n^{\frac{n}{2}}}$, its n-th root is $\sqrt[n]{\left|\frac{1}{\left(\sqrt{i+n}\right)^n}\right|} < \sqrt[n]{\frac{1}{n^{\frac{n}{2}}}} = \frac{1}{\sqrt{n}} < 1$ for $n \geq 2$, $\sqrt[n]{\left|\frac{1}{\left(\sqrt{i+n}\right)^n}\right|} < \sqrt[n]{\frac{1}{n^{\frac{n}{2}}}} = \frac{1}{\sqrt{n}} \to 0$ as $n \to \infty$ (here $0 < 1$), hence, by Cauchy ratio test, the series is convergent.

4) $n^i = e^{i \cdot Ln(n)} = e^{i(\ln(n)+2\pi k i)} = e^{i ln(n) - 2\pi k}$, $\left|n^i\right| = e^{-2\pi k}$ not equal to zero, so $\left|\frac{\overline{1}}{n^i}\right| = e^{2\pi k}$ is a constant, different from zero, then the series is not convergent.