Answer on Question #49484 – Math – Complex Analysis

Find if these sequence convergent of divergent (details)

1) [(1+i)^(1/n)] / square root(n-1)
2) [2^n!] / [2^ (n+1)!]
3) sin ((1+i)/n)

Solution

We say that (z_n) converges to A (write $z_n \to A$ or $\lim_{n\to\infty} z_n = A$), if for every real number ε, there exists a natural number N such that

$$n \geq N \Rightarrow |z_n - A| < \varepsilon$$

1) $\left|\frac{(1+i)^{\frac{1}{n}}}{\sqrt{n-1}}\right| = \frac{2^{\frac{1}{2n}}}{\sqrt{n-1}} \leq \frac{\sqrt{2}}{\sqrt{n-1}} < \frac{2}{\sqrt{n-1}} \to 0$ as $n \to \infty$, besides, $2^{\frac{1}{2n}} \to 2^0 = 1$ as $n \to \infty$, $\sqrt{n-1} \to \infty$ as $n \to \infty$. We take for any $\varepsilon > 0$ natural number $N = \left[\left(\frac{2}{\varepsilon}\right)^2\right] + 1$ such that for all $n \geq N \Rightarrow \frac{2}{\sqrt{n-1}} < \varepsilon$, hence $\left|\frac{(1+i)^{\frac{1}{n}}}{\sqrt{n-1}}\right| < \varepsilon$. Thus, the sequence $z_n = \frac{(1+i)^{\frac{1}{n}}}{\sqrt{n-1}}$ is convergent.

2) $\frac{2^{n!}}{2^{(n+1)!}} = 2^{(n!-(n+1)!)} = 2^{n!(1-(n+1))} = 2^{-n\cdot n!} = \frac{1}{2^{n\cdot n!}} \to 0$ as $n \to \infty$, so the sequence $z_n = \frac{2^{n!}}{2^{(n+1)!}}$ with positive terms is convergent.

3) $\left|sin\left(\frac{1+i}{n}\right)\right| = \left|\frac{e^{i\left(\frac{1+i}{n}\right)} - e^{-i\left(\frac{1+i}{n}\right)}}{2i}\right| \to 0$ as $n \to \infty$, because $\frac{1+i}{n} \to 0$ as $n \to \infty$ (here $\left|\frac{1+i}{n}\right| < \frac{2}{n} \to 0$ as $n \to \infty$). Thus, the sequence $z_n = sin\left(\frac{1+i}{n}\right)$ is convergent.