

Answer on Question #49069 – Math – Statistics and Probability

For each of the situations (#5a-e below) construct the 95% confidence interval estimate to the population. Assume that the results are based on a representative sample of Canadians. Express the estimate in a sentence.

- a) The average age of the 925 respondents was 37.3 years with a standard deviation of 2.9.
- b) Of the 799 respondents questioned, 126 identified themselves as atheist.
- c) Of the 1220 respondents for whom we have information, 222 said that they had been divorced at least once.
- d) Four hundred and thirty two of the 668 respondents questioned said that they are concerned about the environment.
- e) The average number of children for the 945 respondents for which we have information was 1.67 with a standard deviation of .94.

Solution

a) The 95% confidence interval of the mean is

$$\begin{aligned} \left(\mu - z_{\frac{1-\alpha}{2}} \frac{\sigma}{\sqrt{n}}; \mu + z_{\frac{1-\alpha}{2}} \frac{\sigma}{\sqrt{n}} \right) &= \left(37.3 - z_{0.025} \frac{2.9}{\sqrt{925}}; 37.3 + z_{0.025} \frac{2.9}{\sqrt{925}} \right) \\ &= \left(37.3 - 1.96 \frac{2.9}{\sqrt{925}}; 37.3 + 1.96 \frac{2.9}{\sqrt{925}} \right) = (37.11; 37.49). \end{aligned}$$

With 95% confidence level, the mean age of the respondents lies in the interval (37.11; 37.49) years.

b) The 95% confidence interval of the population proportion is

$$\begin{aligned} \left(p - z_{\frac{1-\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}; p + z_{\frac{1-\alpha}{2}} \sqrt{\frac{p(1-p)}{n}} \right) \\ &= \left(\frac{126}{799} - z_{0.025} \sqrt{\frac{\frac{126}{799} \left(1 - \frac{126}{799}\right)}{799}}; \frac{126}{799} + z_{0.025} \sqrt{\frac{\frac{126}{799} \left(1 - \frac{126}{799}\right)}{799}} \right) \\ &= \left(0.158 - 1.96 \sqrt{\frac{0.158(1-0.158)}{799}}; 0.158 + 1.96 \sqrt{\frac{0.158(1-0.158)}{799}} \right) \\ &= (0.133; 0.183). \end{aligned}$$

With 95% confidence level, the proportion of the respondents who identified themselves as atheist lies in the interval (0.133; 0.183).

d) The 95% confidence interval of the population proportion is

$$\begin{aligned}
& \left(p - z_{\frac{1-\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}; p + z_{\frac{1-\alpha}{2}} \sqrt{\frac{p(1-p)}{n}} \right) \\
&= \left(\frac{432}{668} - z_{0.025} \sqrt{\frac{\frac{432}{668} \left(1 - \frac{432}{668}\right)}{668}}; \frac{432}{668} + z_{0.025} \sqrt{\frac{\frac{432}{668} \left(1 - \frac{432}{668}\right)}{668}} \right) \\
&= \left(0.647 - 1.96 \sqrt{\frac{0.647(1-0.647)}{668}}; 0.647 + 1.96 \sqrt{\frac{0.647(1-0.647)}{668}} \right) \\
&= (0.611; 0.683).
\end{aligned}$$

With 95% confidence level, the proportion of the respondents who had been divorced at least once lies in the interval (0.611; 0.683).

c) The 95% confidence interval of the population proportion is

$$\begin{aligned}
& \left(p - z_{\frac{1-\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}; p + z_{\frac{1-\alpha}{2}} \sqrt{\frac{p(1-p)}{n}} \right) \\
&= \left(\frac{222}{1220} - z_{0.025} \sqrt{\frac{\frac{222}{1220} \left(1 - \frac{222}{1220}\right)}{1220}}; \frac{222}{1220} + z_{0.025} \sqrt{\frac{\frac{222}{1220} \left(1 - \frac{222}{1220}\right)}{1220}} \right) \\
&= \left(0.182 - 1.96 \sqrt{\frac{0.182(1-0.182)}{1220}}; 0.182 + 1.96 \sqrt{\frac{0.182(1-0.182)}{1220}} \right) \\
&= (0.160; 0.204).
\end{aligned}$$

With 95% confidence level, the proportion of the respondents who are concerned about the environment lies in the interval (0.160; 0.204).

e) The 95% confidence interval of the mean is

$$\begin{aligned}
& \left(\mu - z_{\frac{1-\alpha}{2}} \frac{\sigma}{\sqrt{n}}; \mu + z_{\frac{1-\alpha}{2}} \frac{\sigma}{\sqrt{n}} \right) = \left(1.67 - z_{0.025} \frac{0.94}{\sqrt{945}}; 1.67 + z_{0.025} \frac{0.94}{\sqrt{945}} \right) \\
&= \left(1.67 - 1.96 \frac{0.94}{\sqrt{945}}; 1.67 + 1.96 \frac{0.94}{\sqrt{945}} \right) = (1.61; 1.73).
\end{aligned}$$

With 95% confidence level, the average number of children for the 945 respondents for which we have information lies in the interval (1.61; 1.73).