

Answer on Question #48898 – Math – Statistics and Probability

Three numbers are selected at random from the numbers 1 to 100. Find the probability that they are in A.P.?

Solution

The number of ways to pick three numbers from a set of $2n$ consecutively numbers, is

$$\binom{2n}{3} = \frac{2n!}{(2n-3)!3!}$$

If we count the ways to pick the highest and lowest such that they have the same parity, there is a specific number for the middle. If the lowest is 1 or 2, there are $n - 1$ choices, if the lowest is 3 or 4, there are $n - 2$, if the lowest is 5 or 6 there are $n - 3$, so the total number of ways to select three numbers in arithmetic progression is

$$2(n-1) + 2(n-2) + \dots + 2(1) = 2 \sum_{k=1}^{n-1} k = 2 \cdot \frac{(n-1)n}{2} = n(n-1).$$

In our problem $2n = 100 \rightarrow n = 50$.

The probability that three numbers are in A.P. is

$$P = \frac{n(n-1)}{\binom{2n}{3}} = \frac{50(50-1)}{\binom{100}{3}} = \frac{50 \cdot 49 \cdot 3! \cdot 97!}{100!} = \frac{50 \cdot 49 \cdot 6}{100 \cdot 99 \cdot 98} = \frac{1}{66} \approx 0.015.$$