Answer on Question #48443 – Math – Statistics and Probability

Weights of women are normally distributed with mean $\mu = 143$ pounds and standard deviation $\sigma = 29$ pounds. If $n = 36$ women are randomly selected, what is the probability that their mean weight is between 120 and 160 pounds?

Solution

The mean weight is

$\bar{X} = \frac{X_1+X_2+\cdots+X_n}{n}$,

The expectation of mean weight is

$E(\bar{X}) = E(X_1) = \mu = 143,$

The variance of mean weight is

$Var(\bar{X}) = \frac{\sigma^2}{n} = \frac{29^2}{36}.$

The probability that mean weight is between 120 and 160 pounds is

$$P(120 < \bar{X} < 160) = P(\bar{X} < 160) - P(\bar{X} < 120).$$

$$P(\bar{X} < 160) = P(z < z_1),$$

where

$$z_1 = \frac{160 - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{160 - 143}{\frac{29}{\sqrt{36}}} = 3.52.$$

$$P(\bar{X} < 120) = P(z < z_2),$$

where

$$z_1 = \frac{120 - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{120 - 143}{\frac{29}{\sqrt{36}}} = -4.76.$$

Thus

$$P(120 < \bar{X} < 160) = P(z < 3.52) - P(z < -4.76) = 0.99978 - 0.00001 = 0.99977.$$

Answer: $\mathbf{0.99977}$.