

Answer on Question #48438 – Math – Calculus

Alfador produces gear for a racquetball league. It costs him \$0.005 per square centimeter to produce racquetballs for the league. In the past, the league used racquetballs with diameter 4 cm, and he produced 30, 000 racquetballs. However, league regulations now require racquetballs to have diameter 4.05 cm.

Use differentials to estimate the increase in cost that Alfador must suffer in order to produce 30, 000 racquetballs under its new regulations. Should this be an overestimate or an underestimate?

You may use the fact that the surface area of a ball satisfies $S = \pi X^2$, where X is the diameter of the ball.

Solution.

$$\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x} \approx f'(x)$$

In our case $f(x) = 30000 * 0.005 * \frac{\pi x^2}{4} = 37.5\pi x^2$, $f'(x) = 75\pi x$,

$$x = 4, \quad \Delta x = 0.05.$$

Thus, $\Delta f(4) = f'(4)\Delta x = 75\pi * 4 * 0.05 \approx 47.12$.

So, the increase in cost equals \$47.12.

$$\Delta f = f(x + \Delta x) - f(x) = 37.5\pi * (4.05^2 - 4^2) \approx 47.42 > 47.12.$$

Therefore, our estimation underestimate the increase in cost.