Answer on Question #48346 – Math – Calculus:

Give the equation of the line tangent to the curve f(x) = (x^(2) + 1)/(e^(2x)+sqrt(x)) at the point (1,f(1)).

Solution.

Tangent line l to the curve $f(x)$ at the point $(x_0, f(x_0)$ has the following representation:

$$l: y = f'(x_0)(x - x_0) + f(x_0);$$

Hence, $x_0 = 1$. So:

$$f(x_0) = \frac{1^2 + 1}{e^{2\cdot 1} + \sqrt{1}} = \frac{2}{e^2 + 1};$$

$$f'(x_0) = \frac{({x_0}^2 + 1)'\left(e^{2x_0} + \sqrt{x_0}\right) - (x_0^2 + 1)\left(e^{2x_0} + \sqrt{x_0}\right)'}{\left(e^{2x_0} + \sqrt{x_0}\right)^2} =$$

$$= \frac{2x_0\left(e^{2x_0} + \sqrt{x_0}\right) - (x_0^2 + 1)\left(2e^{2x_0} + \frac{1}{2\sqrt{x_0}}\right)}{\left(e^{2x_0} + \sqrt{x_0}\right)^2} = \frac{2(e^2 + 1) - 2\left(2e^2 + \frac{1}{2}\right)}{(e^2 + 1)^2} = \frac{1 - 2e^2}{(e^2 + 1)^2};$$

$$l: y = \frac{(1 - 2e^2)(x - 1)}{(e^2 + 1)^2} + \frac{2}{e^2 + 1}.$$