Answer on Question #48271 – Math – Calculus

Trace the curve

y = 1 /6(x^3 − 6x^2 + 9x + 6)

Solution.

$$y = \frac{1}{6}(x^3 - 6x^2 + 9x + 6)$$

x – intercepts: $y = 0 \rightarrow x^3 - 6x^2 + 9x + 6 = 0 \ \rightarrow \ x \approx -0.5$, so

$(-0.5; 0)$ is x − intercept.

y – intercept: $x = 0 \rightarrow y(0) = \frac{1}{6}(0^3 - 6 \cdot 0^2 + 9 \cdot 0 + 6) \rightarrow$

$$y = 1, \text{so } (0; 1) \text{ is y} - \text{intercept.}$$

$y(x)$ **is neither even nor odd function, because** $y(-x) \neq y(x), y(-x) \neq -y(x)$.

$y'(x) = 0 \rightarrow \frac{1}{6}(3x^2 - 12x + 9) = 0 \rightarrow \ x^2 - 4x + 3 = 0 \rightarrow \ \ x = 1 \ or \ x = 3.$

Because $y'(x) > 0$ **when** $x < 1$ **or** $x > 3$; $y'(x) < 0$ **when** $1 < x < 3$,

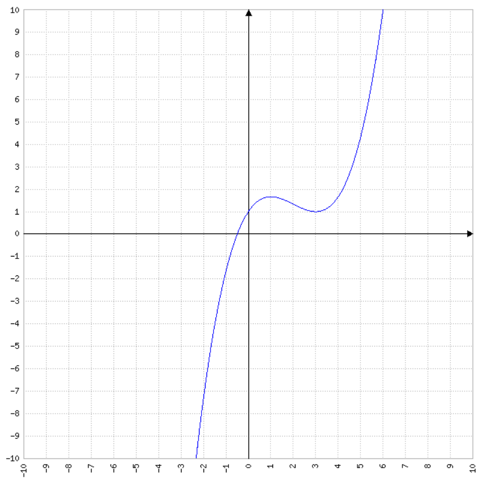
$\left(1, \frac{5}{3}\right)$ **– local maximum.**

$(3, 1)$ **– local minimum.**

Find $y''(x) = \frac{1}{6}(6x - 12) = x - 1$**, hence** $y''(x) > 0$ **when** $x > 1$; $y''(x) < 0$ **when** $x < 1$; $y''(x) = 0$ **when** $x = 1$. **It means that the graph of function** $y = \frac{1}{6}(x^3 - 6x^2 + 9x + 6)$ **is convex upward when** $x < 1$ **and convex downward when** $x > 1$.

When $x \rightarrow -\infty, \ y \rightarrow -\infty \ like \ \frac{x^3}{6}$.

When $x \rightarrow \infty, \ y \rightarrow \infty \ like \ \frac{x^3}{6}$.



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