Answer on Question #48270 – Math – Calculus

Trace the curve
x3 + y3 = 3axy

Solution:

The given equation of the curve can be written as:

$$3axy = x^3 + y^3$$

1. The domain is $\{x \in R\} = (-\infty, \infty)$
2. If the equation of a curve remains unchanged by interchanging x and y, then the curve is symmetrical about the line y = x. Our curve is symmetrical about the line y = x.
3. If there is no constant term in the equation of the curve, the curve passes through origin.

   If the curve is passing through origin, then tangents at the origin can be obtained by equating to zero the lowest degree terms in the equation.

$$3axy = 0$$

$$x^3 + y^3 = 0$$

$$x = 0; y = 0$$

   If there are two tangents to the curve at the origin, the origin is called a double point. Further, If these tangents are real and distinct then the origin is called node

4. x-interept: Put $y = 0$ in the equation to find the x-intercept where the curve intersects the x-axis:

   $x^3 = 0,\ x = 0$

   y-intercept: Put x = 0 in the equation to find the y-intercept where the curve intersects the y-axis;

   $y^3 = 0,\ y = 0$

5. Implicit differentiation applied to equation results in

$$3a\frac{dy}{dx} + 3ay = 3x^2 + 3y^2\frac{dy}{dx}$$

   In solving for $\frac{dy}{dx}$ we get

$$(3y^2 - 3a)\frac{dy}{dx} = 3ay - 3x^2$$

$$\frac{dy}{dx} = \frac{3ay - 3x^2}{3y^2 - 3a} = \frac{ay - x^2}{y^2 - a}$$

Since f ′(x) < 0 when x < 0 , it decreases on (−∞, 0)

6. Local maximum and minimum values : To find critical points, compute f ′(x) and equate it to zero at which y may be local maximum or local minimum.

$$\frac{ay-x^2}{y^2-a} = 0,\ ay - x^2 = 0, y = \frac{x^2}{a}$$

7. Asymptotes : out curve hasn't vertical asymptote but it has slant asymptote. Suppose $y = mx + c$ is a slant asymptote of the given curve.

The highest degree terms in the given equation □3$axy = x^3 + y^3$ are $x^3 + y^3$ of degree three and second highest degree term is 1. Replacing y by m and x by 1 in above terms, we have

Hence, slant asymptote of the curve is ($m = 1; c = a$)

$$x + y + c = 0$$

8. Using above information, we finish the sketch in the following figure.