Answer on Question #48254 – Math - Calculus

A price p (in dollars) and demand x for a product are related by

2x^2−3xp+50p^2=23600.

If the price is increasing at a rate of 2 dollars per month when the price is 20 dollars, find the rate of change of the demand

Solution.

$2x^2 - 3xp + 50p^2 = 23600.$

If $p = 20$, then $2x^2 - 60x + 20000 = 23600 \rightarrow x^2 + 30x - 1800 = 0 \rightarrow$

$\rightarrow x = -60 \text{ or } x = 30$.

The value we select has to be greater than 0, so $x = 30$.

Now, take the derivative of $2x^2 - 3xp + 50p^2 = 23600$.

$\frac{d}{dt}(2x^2 - 3xp + 50p^2) = \frac{d}{dt}(23600).$

$4xx' - 3x'p - 3xp' + 100pp' = 0 \rightarrow x' = -\frac{(-3x+100p)p\prime}{4x-3p}.$

When $p = 20, p' = 2, x = 30$, $x' = -\frac{(-90+2000)2}{40+90} = -\frac{3820}{130} \approx -29.38$.

Rate of change of the demand is approximately equal to -29.38.