Answer on Question #48251 - Math – Calculus

Suppose that for a company manufacturing calculators, the cost, and revenue equations are given by $C = 70000 + 40x$, $R = 400 - \frac{x^2}{20}$, where the production output in one week is x_0 calculators. If the production rate is increasing at a rate of $\Delta x = 500$ calculators when the production output is 6000 calculators, find each of the following: Rate of change in cost(Δ_C), rate of change in revenue(Δ_R), rate of change in profit(Δ_P).

Solution

1. Profit (P): $P = R - C$, where $R \geq 0, C \geq 0$.
2. $x_1 = x_0 + \Delta x$; $x_n = x_0 + n\Delta x$; $x_{n+1} = x_n + \Delta x$;
3. $\Delta_{C_n} = C_{n+1} - C_n = \left(70000 + 40(x_n + \Delta x)\right) - (70000 + 40x_n) = 40\Delta x = 20000$;
4. $\Delta_{R_n} = R_{n+1} - R_n = \left(400 - \frac{(x_n+\Delta x)^2}{20}\right) - \left(400 - \frac{x_n^2}{20}\right) = -\frac{\Delta x}{20}(2x_n + \Delta x) =$
$-\frac{\Delta x}{20}(2x_0 + (2n+1)\Delta x) = -25(12000 + (2n+1)500) = -312500 - 25000n$;
5. $\Delta_{P_n} = P_{n+1} - P_n = R_{n+1} - C_{n+1} - R_n + C_n = \Delta_{R_n} - \Delta_{C_n} = -332500 - 25000n$;

Attention: $R \geq 0$, hence equation $R = 400 - \frac{x^2}{20}$ has sense only for $0 \leq x \leq 89$, thus parts of task regarding revenue and profit with those initial values are meaningless.