

Answer on Question #48218 – Math – Algorithms | Quantitative Methods

Q: The equation $x^3 + 2x^2 - 5 = 0$ has a positive real root in the interval (1, 2). Write a fixed point iteration method and show that it converges. Starting with initial approximation $x = 1.5$ find the root of the equation. Perform two iterations.

Solution.

$$x^3 + 2x^2 - 5 = 0 \rightarrow x = f(x) \text{ where } f(x) = \sqrt{\frac{5}{x+2}}.$$

$$f'(x) = -\frac{1}{2} \sqrt{\frac{5}{(x+2)^3}}.$$

$|f'(x)| < 1$ on $[1, 2]$, thus fixed point iteration method converges.

A fixed point iteration method

$$x_{n+1} = f(x_n) \rightarrow x_{n+1} = \sqrt{\frac{5}{x_n + 2}}$$

$$x_0 = 1.5$$

$$x_1 = \sqrt{\frac{5}{1.5 + 2}} = 1.195$$

$$x_2 = \sqrt{\frac{5}{1.195 + 2}} = 1.251$$