Answer on Question #48177 – Math - Matrix | Tensor Analysis

Question.

Solve the following system of equations $\begin{cases} 3x+2y+4z=7 \\ 2x+y+z=7 \\ x+3y+5z=2 \end{cases}$ using Gauss elimination with pivoting. Store the multipliers and also write the pivoting vector.

Solution.

The extended matrix has the form: $\left(\begin{array}{ccc|c} 3 & 2 & 4 & 7 \\ 2 & 1 & 1 & 7 \\ 1 & 3 & 5 & 2 \end{array}\right)$. The pivoting vector is $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$. Use the next row operation: $3R_2 - 2R_1 \to R_2$ and $3R_3 - R_1 \to R_3$. The matrix after the first pivot looks like this: $\left(\begin{array}{ccc|c} 3 & 2 & 4 & 7 \\ 0 & -1 & -5 & 7 \\ 0 & 7 & 11 & -1 \end{array}\right)$. Now the pivoting vector is $\begin{pmatrix} -1 \\ 7 \end{pmatrix}$. Multiply the second row by 7 and add it to the third row. We shall have: $\left(\begin{array}{ccc|c} 3 & 2 & 4 & 7 \\ 0 & -1 & -5 & 7 \\ 0 & 0 & -24 & 48 \end{array}\right)$. So $-24z = 48 \Leftrightarrow z = -2$, $-y - 5z = 7 \Leftrightarrow y = -5z - 7 = 3$, $3x + 2y + 4z = 7 \Leftrightarrow$

$\Leftrightarrow 3x = 7 - 2y - 4z = 9 \Leftrightarrow x = 3.$

Answer. $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ -2 \end{pmatrix}$.