Answer on Question #48173 – Math – Algebra

On graph paper, draw the axes, and the lines y = 12 and x = 6. The rectangle bounded by the axes and these two lines is a pool table with pockets in the four corners. According to the laws of physics, if a ball travels along a line with slope m and strikes the side of the table, it bounces back along a line with slope m.

a. A ball starts at (2, 6) and moves along the line with slope -2 towards the y-axis. Where does it strike the y-axis? What slope does it have after bouncing off the y-axis? Draw the paths on your graph. Give the equations for both parts of the path of the ball.

b. Follow the ball in (a) for two more bounces, drawing the lines on your graph. Give the coordinates of the points where the ball bounces off each side. State the slope of each part of the path. If the ball could move according to these rules forever, would it ever go in a pocket? (Explain).

c. If your ball is at (2, 6) and you want to put it in the pocket at (6, 0) with one bounce, at which point on the y-axis should you aim? Explain and graph the paths. It may be useful to draw a new "pool table" for this.

Solution:

a. We begin our work with the graphing two lines on the coordinate plane. We have a line horizontally through all points where y=12 and a vertical line through all points where x=6. We can represent the obtained graph on the Figure 1.

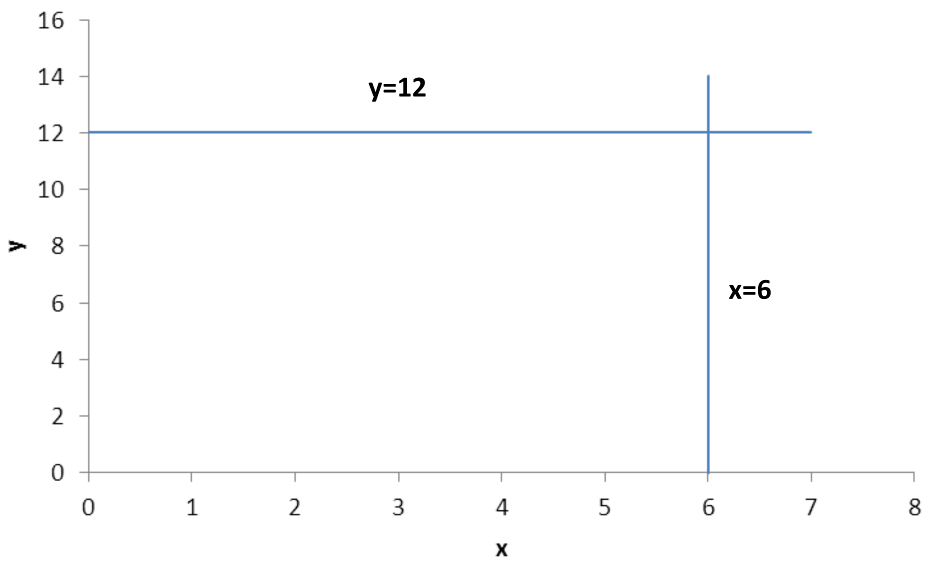

Figure 1 Graph of the two lines y=12 and x=6.

Now we need to consider the question where does a ball strike the y-axis. To find the graphs for the ball just use the point $(2,6)$ and slope $m = -2$ in the form of equation $y = mx + b$. We have to solve for b to find where the ball strikes the y-axis because it is the y-intercept and graph.

We substitute the given coordinates of the point into the formula for determination slope.

$$\frac{y-6}{x-2} = -2$$

Simplify by finding the value of y. We multiply both parts of the equation by (x-2).

$$y - 6 = -2(x - 2)$$

$$y - 6 = -2x + 4$$

We add -6 to both sides of the equation.

$$y = -2x + 10$$

Thus we can note that the equation of line before bouncing will be equal to

$$y = -2x + 10$$

Also we can note that the ball will strike the y-axis at point (0,10) (other words, when the value of x is equal to 0) and the slope after bounce back will be 2 according to the laws of physics, if a ball travels along a line with slope m and strikes the side of the table, it bounces back along a line with slope m.

So we can write the equation of line after bouncing will be equal to

$$\frac{y-10}{x-0} = 2$$

Simplify our equation to find the value of y and write the interested us equation.

$$y - 10 = 2(x - 0)$$

Simplify the right side of the equation by opening the parenthesis.

$$y - 10 = 2x$$

Than we add -2 to both sides of the equation.

$$y = 2x + 10$$

So, we find the equation of line after bouncing which is equal to $y = 2x + 10$.

The result of our calculation we can represent the graph on the Figure 2.

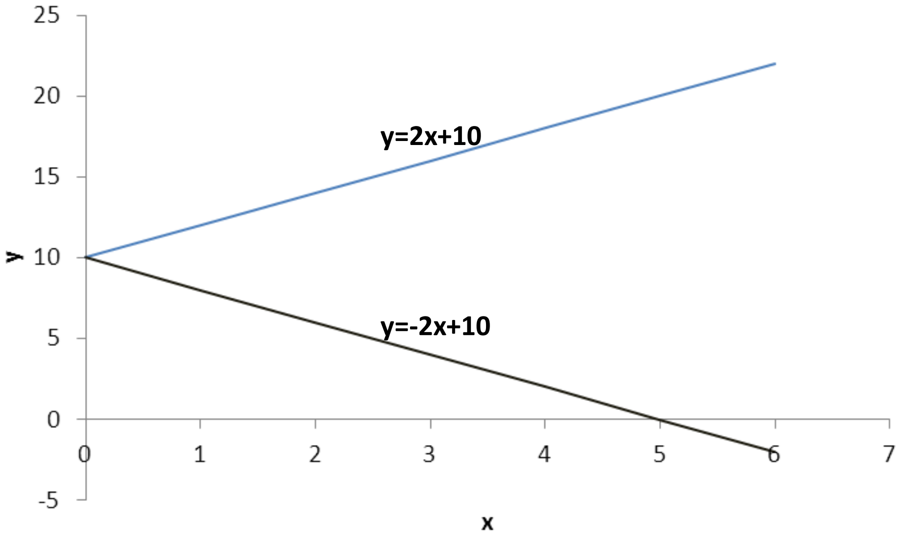

Figure 2 Equations before and after bouncing.

b. Our next task is to find the coordinates at second bounce. We consider the value of y = 0 and substitute into the obtained equation.

$$y = 2x + 10$$

We put the value of y=0 and obtained the following result.

$$2x + 10 = 0$$

We add -10 to both sides of the equation.

$$2x = -10$$

Now we divide both sides of the equation by 2 and find the value of x.

$$x = -5$$

The second bounce will be at point (-5,0).

Now we can find the equation of the line based on the point coordinates.

We know that the equation of the line is in the form

$$y = -2x + b$$

$$0 = (-2)(-5) + b$$

$$0 = 10 + b$$

The value of b will be equal to b=-10.

$$y = -2x - 10$$

Then we need find the equation of the line after 3rd bounce.

We obtained the equation in previous part. So, we can write that the third bounce point of strike on y-axis $y = -2x - 10$ at $x = 6$.

Substitute the value of x into the equation.

$$y = -2(6) - 10$$

Simplify our equation.

$$y = -12 - 10$$

$$y = -22$$

The third bounce point is (6,-22). Now we can find the equation of the line.

$$y = 2x - 22$$

So, we can note that the equation of the line after third bounce will be equal to

$$y = 2x - 22$$

The third bounce will be at point (6,-22).

If only be based on the two equations obtained and coordinates of the points of bounce, we can note that in this case the ball falls into the pocket. Based on these lines (y=12 and x=6), we can note coordinates of pockets that should correspond to our graph specified in Figure 1. These coordinates are (0.0), (0,12), (6,12),(6,0).

c. Now we need to consider last problem. We need to find the point of strike on the y-axis for the ball to fall into (6,0) from (2,6) is (0,y). Let the slope before strike be m and the slope after strike be -m then we can write the following.

$$\frac{y-6}{x-2} = m$$

We note the second case.

$$\frac{y-0}{x-6} = -m$$

We can write the following equity.

$$\frac{y-6}{x-2} = -\frac{y}{x-6}$$

Simplify by multiplying left and right side of the equation.

$$(y-6)(x-6) = -y(x-2)$$

Now we simplify by opening the parenthesis.

$$xy - 6y - 6x + 36 = -yx + 2y$$

Combine like terms on the left and right sides of the equation.

$$xy + yx - 6y - 6x - 2y = -36$$

$$2xy - 8y - 6x = -36$$

As we noted earlier the we need to find the point of strike on the y-axis for the ball to fall into (6,0) from (2,6) is (0,y). Substitute the coordinate of point equal to (0,y) into the obtained equation.

$$-8y = -36$$

Now we divide both sides of the equation by -8 we obtained the following result.

$$y = 4.5$$

Thus we can conclude that coordinate of the point will be equal to (0,4.5).

The result of our calculations is represented on the Figure 3.

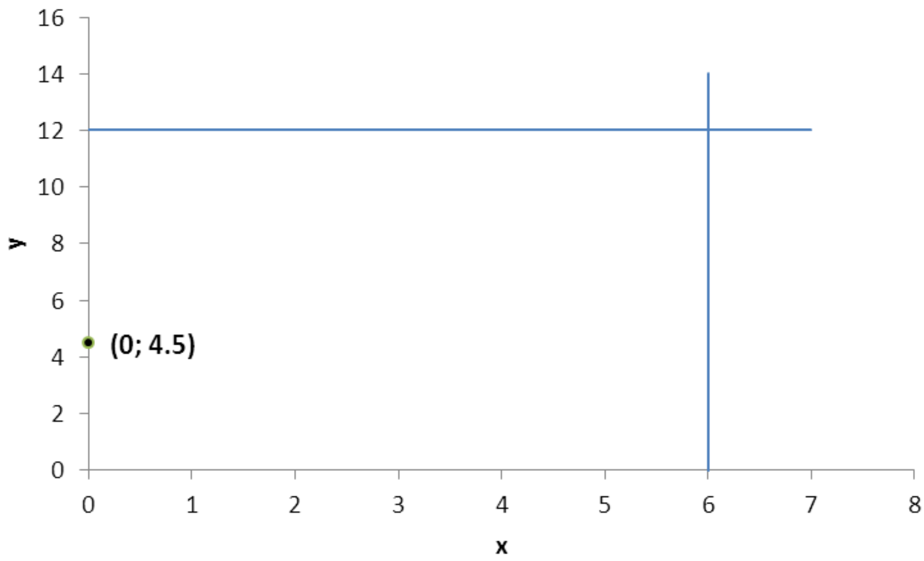

Figure 3 The point of strike on the y-axis for the ball to fall into (6,0) from (2,6) is (0,y).